EUSFLAT 2023 - AGOP 2023

13th Conference of the European Society for Fuzzy Logic

12th International Summer School on Aggregation Operators

September 4-8, Palma, Spain

BOOK OF ABSTRACTS

Book of Abstracts

EUSFLAT 2023 and AGOP 2023

September 4–8, Palma, Spain

13th Conference of the European Society for Fuzzy Logic
12th International Summer School on Aggregation Operators

Editors: Sebastia Massanet, Susana Montes, Daniel Ruiz-Aguilera and Manuel González-Hidalgo

Book of Abstracts

EUSFLAT 2023 and AGOP 2023

September 4–8, Palma, Spain

13th Conference of the European Society for Fuzzy Logic
12th International Summer School on Aggregation Operators

Editors: Sebastia Massanet, Susana Montes, Daniel Ruiz-Aguilera and Manuel González-Hidalgo

First Edition: September, 2023
Edited by: SCOPIA Research Group. Universitat de les Illes Balears.
Campus Universitari
Ctra. de Valldemossa, Km 7.5, E-07122 Palma (Illes Balears), Spain
http://scopia.uib.eu
ISBN: 978-84-09-52808-0
DL: PM 01076-2023
Printed in Spain

Preface

Almost 24 years ago, the 1999 EUSFLAT-ESTYLF Joint Conference was held in Palma. This conference, which took place from September 22 to 25, 1999, was organized by the University of the Balearic Islands and the European Society for Fuzzy Logic and Technology (EUSFLAT) and it was the first edition of the conferences of this society, after its foundation that same year. After the success of the first edition, this conference has been organized every two years in many European towns. Namely, Leicester (United Kingdom), Zittau (Germany), Barcelona and Gijon (Spain), Ostrava and Prague (Czech Republic), Lisbon (Portugal), Aix-Les-Bains (France), Milano (Italy), Warsaw (Poland) and Bratislava (Slovak Republic) have been the venue for subsequent editions. Now, on the eve of the 25th anniversary, it is time for the EUSFLAT conference to return to its origins, back to its roots.

The world has changed a lot since 1999. However, some facts remain stable. The aim of the conference, in line with the mission of the EUSFLAT Society, is to bring together theoreticians and practitioners working on fuzzy logic, fuzzy systems, soft computing, and related areas and to provide for them a platform for the exchange of ideas, discussing newest trends and networking. During these years and due to the successful development of fuzzy logic and the corresponding technology, interest in fuzzy logic has been growing steadily, and the EUSFLAT conference has been the main European conference in this scientific field. However, despite being a predominantly European conference, many researchers from other continents attend the EUSFLAT conferences edition after edition, recognizing that they constitute a reference point every two years for important advances in the lines of research associated with this field. In the specific case of the Balearic Islands, it should be noted that since the late 1980s an intense research in fuzzy logic has been developed within the framework of the research group led by Gaspar Mayor and Joan Torrens, who are now happily retired. The new generation took the baton and the responsibility of organizing this edition of the EUSFLAT conference.

This 2023 edition of the EUSFLAT conference was co-located for the second time with two traditional events, namely with AGOP 2023 - International Summer School on Aggregation Operators; and with FQAS 2023 - International Conference on Flexible Query Answering Systems. We would like to express our thanks to the management of these events for sharing the vision of the joint multiconference. Special mention should be given to the AGOP summer school, with which these proceedings are shared. The AGOP summer school is organized biannually by the AGOP working group of EUSFLAT, reaching this year its 12th edition after its birth in 2001 in Oviedo (Spain). This event focuses on aggregation functions, a family of operators which have numerous applications, including, but not limited to, data fusion, statistics, image processing and decision making.

Therefore, this volume constitutes the book of abstracts of the 13th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT) and the 12th International Summer School on Aggregation Operators (AGOP). The works included in the book of abstracts have been subject to a thorough review process by at least two highly qualified peer reviewers, by using a single-blind process. The volume contains very attractive and up-to-date topics in fuzzy logic and related fields, which will result in significant interest of the international research communities active in the covered areas. Special gratitude is due to the extremely relevant role of the organizers of the special sessions. Thanks to their vision and hard work, we have been able to collect many papers on focused topics which we are sure will result, during the conference, in very interesting presentations and stimulating discussions at the sessions. It should be noted that for EUSFLAT and AGOP 2023, 71 full papers and 90 abstracts (161 submissions in total) were submitted from which 61 full papers have been accepted.

Finally, we would like to express our gratitude to all chairs and the organizing team for making these conferences possible. We believe that we will experience an excellent and unforgettable conference. We hope that you enjoyed it and that it brought home many new fruitful ideas for your research, and also that you enjoyed this beautiful island, Mallorca, the largest island in the Balearic Islands, set in of the Mediterranean Sea, with its great beaches, amazing atmosphere and cultural richness.

September 2023

Sebastia Massanet
Susana Montes
Daniel Ruiz-Aguilera
Manuel González-Hidalgo

Organization

EUSFLAT 2023 and AGOP 2023 were organized by the research group in Soft Computing, Image Processing and Aggregation (SCOPIA), from the Department of Mathematics and Computer Science of the University of the Balearic Islands (UIB), in cooperation with the EUSFLAT (European Society for Fuzzy Logic and Technology).

General Chairs

Sebastia Massanet	Universitat de les Illes Balears, Spain
Susana Montes	Universidad de Oviedo, Spain
Daniel Ruiz-Aguilera	Universitat de les Illes Balears, Spain

Organizing Chair

Manuel González-Hidalgo	Universitat de les Illes Balears, Spain

EUSFLAT Programme Chairs

Humberto Bustince	Universidad Pública de Navarra - UPNA, Spain
Vladik Kreinovich	University of Texas at El Paso, USA

Publication Chairs

Luis Martínez Lopez	Universidad de Jaén, Spain
Juan Vicente Riera	Universitat de les Illes Balears, Spain

Publicity Chairs

Przemyslaw Grzegorzewski	Warsaw University of Technology, Poland
Balasubramaniam Jayaram	Indian Institute of Technology Hyderabad, India
Manuel Ojeda Aciego	Universidad de Málaga, USA
Gabriella Pasi	Università degli Studi di Milano-Bicocca, Italy

Special Sessions Chairs

Michal Baczyński	University of Silesia in Katowice, Poland
Luis Magdalena	Universidad Politécnica de Madrid, Spain
Peter Sussner	University of Campinas, Brazil

Grants and Awards Chairs

Christophe Marsala	Université Pierre et Marie Curie, France
Javier Montero	Universidad Complutense de Madrid, Spain
Eulalia Szmidt	Polish Academy of Sciences, Poland

Organizing Committee

Isabel Aguiló	Universitat de les Illes Balears, Spain
Pedro Bibiloni	Universitat de les Illes Balears, Spain
Raquel Fernandez-Peralta	Universitat de les Illes Balears, Spain
Manuel González-Hidalgo	Universitat de les Illes Balears, Spain

Sebastia Massanet	Universitat de les Illes Balears, Spain
Arnau Mir	Universitat de les Illes Balears, Spain
Marc Munar	Universitat de les Illes Balears, Spain
Juan Vicente Riera	Universitat de les Illes Balears, Spain
Daniel Ruiz-Aguilera	Universitat de les Illes Balears, Spain
Lidia Talavera-Martínez	Universitat de les Illes Balears, Spain
Llorenç Valverde	Universitat de les Illes Balears, Spain

SS1: Interval uncertainty. Organizers

Martine Ceberio	University of Texas at El Paso, USA
Vladik Kreinovich	University of Texas at El Paso, USA

SS2: Information fusion techniques based on aggregation functions, preaggregation functions and their generalizations. Organizers

Humberto Bustince	Universidad Pública de Navarra - UPNA, Spain
Graçaliz Pereira Dimuro	Universidade Federal do Rio Grande, Brazil
Javier Fernández	Universidad Pública de Navarra - UPNA, Spain
Tiago da Cruz Asmus	Universidade Federal do Rio Grande, Brazil

SS3: Evaluative linguistic expressions, generalized quantifiers and applications. Organizers

Vilém Novák	University of Ostrava, Czech Republic
Petra Murinová	University of Ostrava, Czech Republic
Stefania Boffa	University of Milano-Bicocca, Italy

SS4: Neural networks under uncertainty and imperfect information. Organizers

Humberto Bustince	Universidad Pública de Navarra - UPNA, Spain
Javier Fernández	Universidad Pública de Navarra - UPNA, Spain
Iosu Rodríguez-Martínez	Universidad Pública de Navarra - UPNA, Spain
Mikel Ferrero-Jaurrieta	Universidad Pública de Navarra - UPNA, Spain
Jonata Wieczynski	Universidad Pública de Navarra - UPNA, Spain

SS5: Imprecision modeling and management in XAI systems. Organizers

Ciro Castiello	Università degli Studi di Bari Aldo Moro, Italy
Marie-Jeanne Lesot	Sorbonne Université, France
Corrado Mencar	Università degli Studi di Bari Aldo Moro, Italy

SS6: Recent trends in mathematical fuzzy logics. Organizers

Stefano Aguzzoli	Università degli Studi di Milano, Italy
Brunella Gerla	Università dell'Insubria, Italy

SS7: Fuzzy graph-based models: theory and application. Organizers

Stefan Stanimirović	University of Niš, Serbia
Ivana Micić	University of Niš, Serbia

SS8: New frontiers of computational intelligence for pervasive healthcare systems. Organizers

Gabriella Casalino	University of Bari Aldo Moro, Italy
Giovanna Castellano	University of Bari Aldo Moro, Italy
Uzay Kaymak	Eindhoven University of Technology, The Netherlands
Gianluca Zaza	University of Bari Aldo Moro, Italy

SS9: Fuzzy implication functions. Organizers

Michal Baczyński	University of Silesia in Katowice, Poland
Balasubramaniam Jayaram	Indian Institute of Technology Hyderabad, India
Sebastia Massanet	Universitat de les Illes Balears, Spain

SS10: New challenges and ideas in statistical inference and data analysis. Organizers

Przemyslaw Grzegorzewski	Warsaw University of Technology, Poland
Katarzyna Kaczmarek-Majer	Polish Academy of Sciences, Poland

SS12: Representing and managing uncertainty: different scenarios, different tools. Organizers

Davide Ciucci	University of Milano-Bicocca, Italy
Chris Cornelis	Ghent University, Belgium
Jesús Medina	University of Cádiz, Spain
Dominik Slezak	University of Warsaw, Poland

Program Committee

Aguiló, Isabel	Universitat de les Illes Balears, Spain
Akbarzadeh T. M. R.	Ferdowsi University of Mashhad, Iran
Acampora, Giovanni	Università degli Studi di Napoli Federico II, Italy
Aguzzoli, Stefano	University of Milan, Italy
Aliev, Rafik Aziz	Azerbaijan State Oil and Industry University, Azerbaijan
Allahviranloo, Tofigh	İstinye University, Turkey
Alonso, Jose Maria	Universidad de Santiago de Compostela, Spain
Asmus, Tiago da Cruz	Universidade Federal do Rio Grande, Brazil
Baczyński, Michal	University of Silesia in Katowice, Poland
Batyrshin, Ildar	Instituto Politécnico Nacional, Mexico
Bloch, Isabelle	Sorbonne Université, CNRS, France
Bobillo, Fernando	Universidad de Zaragoza, Spain
Boffa, Stefania	University of Milano-Bicocca, Italy
Borisov, Vadim V.	Moscow Power Engineering Institute, Russia
Bouchon-Meunier, Bernadette	Sorbonne Université, France
Burczyński, Tadeusz S.	Polish Academy of Sciences, Poland
Burda, Michal	University of Ostrava, Czech Republic
Bustince, Humberto	Universidad Pública de Navarra, Spain
Cabrera, Inmaculada	Universidad de Málaga, Spain
Calvo, Tomasa	Universidad de Alcalá, Spain
Carlsson, Christer	Institute for Advanced Management Systems Research, Finland
Carvalho, Joao Paulo	Universidade de Lisboa, Portugal
Casalino, Gabriella	Università degli studi di Bari Aldo Moro, Italy
Castellano, Giovanna	Università degli studi di Bari Aldo Moro, Italy
Castiello, Ciro	Università degli studi di Bari Aldo Moro, Italy
Castillo, Oscar	Tijuana Institute of Technology, Mexico
Ceberio, Martine	University of Texas at El Paso, USA
Chalco-Cano, Yurilev	Universidad de Tarapacá, Chile

Chountas, Panagiotis	University of Westminster, UK
Ciucci, Davide	University of Milano-Bicocca, Italy
Collan, Mikael	Lappeenranta-Lahti University of Technology, Finland
Cordero, Pablo	Universidad de Málaga, Spain
Cordón, Oscar	Universidad de Granada, Spain
Cornelis, Chris	Ghent University, Belgium
D'Aniello, Giuseppe	University of Salerno, Italy
Dankova, Martina	University of Ostrava, Czech Republic
De Baets, Bernard	Ghent University, Belgium
De Tré, Guy	Ghent University, Belgium
Di Nola, Antonio	Università degli Studi di Salerno, Italy
Diaz, Irene	Universidad de Oviedo, Spain
Dick, Scott	University of Alberta, Canada
Dimuro, Graçaliz	Universidade Federal do Rio Grande, Brazil
Dubois, Didier	Centre National de la Recherche Scientifique, France
Durante, Fabrizio	Università del Salento, Italy
Dvorak, Antonin	University of Ostrava, Czech Republic
Ekel, Petr	Pontifícia Universidade Católica de Minas Gerais, Brasil
Fernandez, Javier	Universidad Pública de Navarra, Spain
Ferrero-Jaurrieta, Mikel	Universidad Pública de Navarra, Spain
Figueroa-García, Juan C.	Universidad Distrital Francisco José de Caldas, Colombia
Gaeta, Matteo	Università degli Studi di Salerno, Italy
Gagolewski, Marek	Deakin University, Australia
Garibaldi, Jonathan M.	University of Nottingham, UK
Gerla, Brunella	Università degli Studi dell'Insubria, Italy
Godo, Lluis	Artificial Intelligence Research Institute, IIIA - CSIC, Spain
Gómez-Romero,Juan	Universidad de Granada, Spain
Gomide, Fernando	Universidade Estadual de Campinas, Brazil
González-Hidalgo, Manuel	Universitat de les Illes Balears, Spain
Grzegorzewski, Przemyslaw	Warsaw University of Technology, Poland
Hadjali, Allel	University of Poitiers, France
Halaš, Radomír	Palacký University Olomouc, Czech Republic
Hirota, Kaoru	Tokyo Institute of Technology, Japan
Holčapek, Michal	University of Ostrava, Czech Republic
Holena, Martin	Czech Academy of Sciences, Czech Republic
Homenda, Wladyslaw	Warsaw University of Technology, Poland
Honda, Aoi	Kyushu Institute of Technology, Japan
Hudec, Miroslav	University of Economics in Bratislava, Slovakia
Hurtik, Petr	University of Ostrava, Czech Republic
James, Simon	Deakin University, Australia
Janiš, Vladimir	Matej Bel University, Slovakia
Jayaram, Balasubramaniam	Indian Institute of Technology Hyderabad, India
Jin, Lesheng	Nanjing Normal University, China
Kaczmarek-Majer, Katarzyna	Polish Academy of Sciences, Poland
Kahraman, Cengiz	Istanbul Technical University, Turkey
Kalina, Martin	Slovak University of Technology, Slovakia
Kaymak, Uzay	Eindhoven University of Technology, The Netherlands
Kerre, Etienne	Ghent University, Belgium
Kim, Sungshin	Pusan National University, South Korea
Klawonn, Frank	Ostfalia University of Applied Sciences, Germany
Klement, Erich Peter	Johannes Kepler University Linz, Austria
Koczy, Laszlo T.	Budapest University of Technology and Economics, Hungary
Kolesárová, Anna	Slovak University of Technology, Slovakia
Konecny, Jan	Palacký University Olomouc, Czech Republic
Kreinovich, Vladik	University of Texas at El Paso, USA
Kupka, Jiří	University of Ostrava, Czech Republic
Lesot, Marie-Jeanne	Sorbonne Université, France

Li, Jun	Communication University of china, China
Liu, Xinwang	Southeast University, China
López-Molina, Carlos	Universidad Pública de Navarra, Spain
Lughofer, Edwin	Johannes Kepler University Linz, Austria
Magdalena Layos, Luis	Universidad Politécnica de Madrid, Spain
Marcelloni, Francesco	University of Pisa, Italy
Marsala, Christophe	Université Pierre et Marie Curie, France
Martín-Bautista, María José	Universidad de Granada, Spain
Martínez, Luis	Universidad de Jaén, Spain
Massanet, Sebastia	Universitat de les Illes Balears, Spain
Medina, Jesús	Universidad de Cádiz, Spain
Mencar, Corrado	University of Bari Aldo Moro, Italy
Mendel, Jerry	University of Southern California, USA
Mesiar, Radko	Slovak University of Technology, Slovakia
Mesiarová-Zemánková, Andrea	University of Ostrava, Slovakia
Michalíková, Alžbeta	Matej Bel University, Slovakia
Micić, Ivana	University of Niš, Serbia
Mir, Arnau	Universitat de les Illes Balears, Spain
Mnasri, Zied	Université de Tunis-El Manar, Tunisia
Močkoř, Jiří	University of Ostrava, Czech Republic
Montero, Javier	Universidad Complutense de Madrid, Spain
Montes, Susana	Universidad de Oviedo, Spain
Murinová, Petra	University of Ostrava, Czech Republic
Navara, Mirko	Czech Technical University, Czech Republic
Nguyen, Phuong	Thang Long University, Vietnan
Noguera, Carles	University of Siena, Italy
Novák, Vilém	University of Ostrava, Czech Republic
Nurmi, Hannu	University of Turku, Finland
Ojeda-Aciego, Manuel	Universidad de Málaga, Spain
Olivas, José Angel	Universidad de Castilla-La Mancha, Spain
Pérez-Fernández, Raúl	Universidad de Oviedo, Spain
Perfilieva, Irina	University of Ostrava, Czech Republic
Petrík, Milan	Czech University of Life Sciences Prague, Czech Republic
Pivert, Oliver	University of Rennes, France
Portmann, Edy	University of Fribourg, Switzerland
Prade, Henri	Centre national de la recherche scientifique, France
Reformat, Marek	University of Alberta, Canada
Riera, Juan Vicente	Universitat de les Illes Balears, Spain
Rodríguez-Martínez, Iosu	Universidad Pública de Navarra, Spain
Romero, Francisco P.	Universidad de Castilla-La Mancha, Spain
Rovetta, Stefano	University of Genoa, Italy
Ruiz-Aguilera, Daniel	Universitat de les Illes Balears, Spain
Ruiz, María Dolores	Universidad de Granada, Spain
Sadeghian, Alireza	Toronto Metropolitan University, Canada
Sanchez, Daniel	Universidade Estadual de Campinas, Brazil
Seising, Rudolf	Deutsches Museum, Germany
Serrano-Guerrero, Jesús	Universidad de Castilla-La Mancha, Spain
Seselja, Branimir	University of Novi Sad, Serbia
Sessa, Salvatore	Università degli Studi di Napoli Federico II, Italy
Skowron, Andrzej	University of Warsaw, Poland
Slezak, Dominik	University of Warsaw, Poland
Sostak, Alexandre	University of Latvia, Latvia
Sousa, João Miguel da Costa	Universidade de Lisboa, Portugal
Stanimirović, Stefan	University of Niš, Serbia
Stefanini, Luciano	Urbino University, Italy
Štěpnička, Martin	University of Ostrava, Czech Republic

Straccia, Umberto	Istituto di Scienza e di Tecnologie dell'Informazione, Italy
Stupňanová, Andrea	Slovak University of Technology, Slovakia
Sussner, Peter	Universidade Estadual de Campinas, Brazil
Szmidt, Eulalia	Polish Academy of Sciences, Poland
Tabacchi, Marco Elio	Universita degli Studi di Palermo, Italy
Takáč, Zdenko	Slovak University of Technology, Slovakia
Torra, Vicenç	Umeå University, Sweden
Tsai, Ching-Chih	National Chung Hsing University, Taiwan
Tulupyev, Alexander	Saint Petersburg State University, Russia
Verdegay, José Luis	Universidad de Granada, Spain
Vetterlein, Thomas	Johannes Kepler University Linz, Austria
Wang, Lipo	Nanyang Technological University, Singapore
Watada, Junzo	Waseda University, Japan
Wieczynski, Jonata	Universidad Pública de Navarra, Spain
Wilbik, Anna	Maastricht University, The Netherlands
Yarushkina, Nadejda	Ulyanovsk State Technical University, Russia
Ying, Hao	Wayne State University, USA
Yoon, Jin Hee	Sejong University, South Korea
Zadrożny, Sławomir	Systems Research Institute, Poland
Zaza, Gianluca	Università degli studi di Bari Aldo Moro, Italy

Additional Reviewers

Akhtar, Jamil
Alijani, Zahra
Ben Souissi, Souhir
Bianchi, Matteo
Boeckling, Toon
Cao, Nhung
Cruz, Anderson
Csato, Laszlo
De Miguel, Laura
Dyba, Martin
Fechner, Włodzimierz
Felix, Rudolf
Fernández Sánchez, Juan
Fernandez-Peralta, Raquel
Fiala, Karel
Flaminio, Tommaso
Franco, Carlos
Fumanal Idocin, Javier
Guarino, Alfonso
Hongjun, Zhou
Kmita, Kamil
Kumar Gupta, Vikash
Labroche, Nicolas
Lapenta, Serafina
Linh, Nguyen
Lobo, David
Madrid, Nicolas
Mandal, Sayantan
Milosevic, Pavle
Miś, Katarzyna
Munar, Marc
Murinova, Petruska
Nanavati, Kavit
Paiva, Rui
Paseka, Jan
Pazienza, Andrea
Peralta, Daniel
Picerno, Pietro
Pocs, Jozef
Poledica, Ana
Pra Baldi, Michele
Rao Vemuri, Nageswara
Rijcken, Emil
Rodríguez, Iosu
Romaniuk, Maciej
Rubio-Manzano, Clemente
Santoro, Domenico
Santos, Helida
Scaringi, Raffaele
Schicchi, Daniele
Siudem, Grzegorz
Spolaor, Simone
Tepavcevic, Andreja
Torrens, Adrià
Ubeda, Manuel
Vittaut, Jean-Noël
Yu, Peng
Żogała-Siudem, Barbara

Sponsoring Institutions

EUSFLAT (European Society for Fuzzy Logic and Technology)
Vicerectorat de Política Científica i Investigació, UIB
Department of Mathematics and Computer Science, UIB
Palma Town Council
Conselleria de Fons Europeus, Universitat i Cultura, Balearic Islands Government
MCIN (Ministry of Science and Innovation), R+D+i Project PID2020-113870GB-I00 funded by MCIN/AEI/10.13039/501100011033/.

Table of Contents

III AGOP General Track

IV SS1: Interval uncertainty

V SS2: Information fusion techniques based on aggregation functions, preaggregation functions and their generalizations

VI SS3: Evaluative linguistic expressions, generalized quantifiers and applications

VII SS4: Neural networks under uncertainty and imperfect information

VIII SS5: Imprecision modeling and management in XAI systems

IX SS6: Recent trends in mathematical fuzzy logics

X SS7: Fuzzy graph-based models: theory and application

XI SS9: Fuzzy implication functions

Part I

Invited Talks

Construction and representation of associative functions

Andrea Mesiarová-Zemánková[1,2]

[1] Mathematical Institute, Slovak Academy of Sciences, Štefánikova 49, 814 73 Bratislava, Slovakia
[2] Institute for Research and Applications of Fuzzy Modelling, University of Ostrava, CE IT4Innovations, 30. dubna 22, 70103 Ostrava, Czech Republic

The ordinal sum and lately also the z-ordinal sum are construction methods widely used for construction and representation of associative aggregation functions such as t-norms, t-conorms, uninorms, nullnorms and n-uninorms. The ordinal sum (z-ordinal sum) is determined by the associated linear (partial) order which corresponds to a commutative, associative, internal (idempotent) function on the related index set. The commutativity of this function implies that although the (z-)ordinal sum can be used also for construction of non-commutative associative functions, this non-commutativity is a property of the corresponding summands rather than of the construction itself. After a brief summary of results related to construction and representation of associative aggregation functions constructed by (z-)ordinal sum construction we will discuss the very recent research which shows that construction methods with non-commutative nature can be based on non-commutative, associative, internal (idempotent) functions defined on the related index set which can be represented by the associated linear (partial) pair-order. Therefore, the corresponding construction method is called the non-commutative ordinal sum. Similarly, as each continuous t-norm, each continuous t-conorm and each (n-)uninorm with continuous underlying functions, defined on a real interval, can be represented as a (z-)ordinal sum of representable or trivial semigroups, each continuous, associative aggregation function, defined on a real interval, i.e., each semi-t-operator, can be represented by a non-commutative ordinal sum of representable or trivial semigroups.

Acknowledgement: This work was supported by grants VEGA 1/0036/23 and APVV-20-0069.

Biography

Andrea Mesiarová-Zemánková received the Ph.D. degree in Applied Mathematics from the Mathematical Institute of Slovak Academy of Sciences and Department of Mathematics and Descriptive Geometry STU in 2005. She worked at Trinity College Dublin as a research assistant and later as an independent researcher. Currently she is a senior researcher at the Mathematical Institute of Slovak Academy of Sciences and at the Institute for Research and Applications of Fuzzy Modelling, University of Ostrava. In 2014 she received Award of Visegrad Group Academies for Young Researchers, in 2011 the Price of the President of Slovak Republic for young scientists and in 2010 the Price of the Ministry of Education, Science, Research and Sport of Slovak Republic.

In 2010 she was awarded by AXA Research Fund Postdoctoral fellowship and in 2013 by Fellowship of Slovak Academy of Sciences. Her main research interests are the aggregation theory, associative functions on bounded lattices, multi-polar aggregation, and non-additive measures and integrals. She has participated in several national and international projects involving various aspects of aggregation. She has served as a program committee member of several international conferences and she is a reviewer of several distinguished journals, such as Fuzzy Sets and Systems, Information Sciences, IEEE Transactions on Fuzzy Systems, International Journal of Approximate Reasoning, Soft Computing, Kybernetika, Mathematica Slovaca, Iranian Journal of Fuzzy Systems, and others.

Identifying Misinformation Online: Open Issues and Challenges

Gabriella Pasi

Department of Computer Science, Systems and Communication (DISCo), University of Milano-Bicocca, Italy

In the World Wide Web and on social media, a large amount of content of different nature and origin is spread without any form of reliable external control; in this context, the risk of running into misinformation is not negligible. In recent years, an increasing awareness of the possible risks of running into fake news, fake reviews, or health misinformation has emerged. This has motivated a considerable amount of research finalized at defining systems that are able to predict if a piece of information is truthful. Several approaches have been proposed in the literature to automatically assess the truthfulness of content disseminated online. Most of them are data-driven approaches, based on machine learning techniques, but recently also model-driven approaches have been studied, based, in particular, on the Multi-Criteria Decision Making (MCDM) paradigm, and also based on the use of Knowledge Bases. Both categories of approaches make use of prior knowledge related to the problem under consideration, which is injected into the decision process. This talk will present an overview of the approaches to coping with the problem of misinformation detection, with particular emphasis on model-driven approaches, their open issues, and current challenges. Among these approaches, those based on the aggregation of salient features related to misinformation will be considered. Their application to specific problems will also be addressed, as well as the problem of evaluating these systems.

Biography

Gabriella Pasi is Professor at the Department of Computer Science, Systems and Communication (DISCo) of the University of Milano-Bicocca, where she leads the Information and Knowledge Representation, Retrieval, and Reasoning (IKR3) research lab. She is also Head of the Department and Vice-Rector for International Relations. Her main research interests include Information Retrieval and Natural Language Processing, Knowledge Representation and Reasoning, User Modelling, and Social Media Analytics. She has been the Program co-Chair of several conferences, among which: ACM ICTIR 2022, CLEF 2022, ECIR 2019, Short Paper Track at SIGIR 2014, and Poster Track at SIGIR 2010. She was co-organizer of ESSIR 2019, held in Milan, at the premises of the University of Milano-Bicocca. She has been Senior Track Chair (Information Retrieval Track) at ACL 2022, and Chair of the Test of Time Award committee at ECIR 2021. She has delivered Keynote talks at several international conferences. She is a member of the Steering Committee of ESSIR. She was a panelist at the Panel "Women in IR" at SIGIR 2022. She is Associate Editor of several international journals, among which ACM Computing Surveys, IEEE Transactions on Fuzzy Systems, and the International Journal of Data Science and Analytics. She is co-director of the ELLIS Unit in Milan (European Laboratory for Learning and Intelligent Systems).

Fuzzy relational compositions as powerful, comprehensible, and easy-to-construct models for distinct purposes

Martin Štěpnička

CE IT4I – IRAFM, University of Ostrava, Ostrava 701 03, Czech Republic

The concept of fuzzy relational compositions, which is a generalization of compositions of classical binary relations, is by far not a recent one. It has been studied in the late 1970s and early 1980s mainly by Wyllis Bandler and Ladislav J. Kohout and shortly after by many other scholars, e.g., Etienne E. Kerre, Witold Pedrycz, Bernard De Baets, Radim Belohlavek, Antonio Di Nola, or Irina Perfilieva. The ability to model instructions formulated in predicate fuzzy logic in an elegant and computationally cheap way made them a good candidate for further development and it probably stayed behind the motivation of the many researchers to investigate them deeply.

Due to the dominance of fuzzy rule-based models among other fuzzy modeling tools at that time, the main direction led naturally to the fuzzy relational inference systems where the inference engine is derived from the particular appropriate composition. However, apart from this direction, the huge potential of the fuzzy relational compositions was, unfortunately, partly unused.

We can only speculate about the reasons. One of them might be hidden in the fact that some of the crucial works of Bandler and Kohout were published in proceedings of local conferences in small quantities and there was no way of spreading them electronically before the internet. The other one might be in certain limitations of the applications of fuzzy relational compositions. For example, the basic composition is based on the existential quantifier, and the other, say standard, compositions - the Bandler-Kohout products, are based on the universal quantifier, and there is a huge gap between these two quantifiers. Another problem is that a single composition cannot capture all that needs to be captured for particular applications.

However, if we dare to enrich the existing knowledge of fuzzy relational compositions with other tools such as generalized quantifiers that can fill the gap between the existential quantifier and the universal quantifier, and if we consider the compositions to be only basic constituents that can be combined to more complicated constructions, we have a key to unlock the door to an exciting journey through the expressive power of fuzzy relational compositions. The unlocked journey leads to further constructions of fuzzy relational compositions built as the combination of the simple ones establishing powerful, comprehensible, and easy-to-construct models for distinct purposes.

Biography

Martin Štěpnička received his habilitation (Docent – Associative Professorship) in Applied Mathematics at the University of Ostrava in 2012. Since March 2023, he is the Vice-rector for Research and Artistic Activities at the University of Ostrava. Beforehand, he served as the Director of the Centre of Excellence IT4Innovations – Institute for Research and Applications of Fuzzy Modeling, University of Ostrava for two years and he held the vice-director and senior researcher position in the preceding years. Martin Štěpnička also held the positions of the President of the European Society for Fuzzy Logic and Technology (EUSFLAT) for two consecutive terms – elected in 09/2017 and re-elected in 09/2019. He is also a Senior IEEE (CIS) Member.

He is an area Editor of JCR journals Fuzzy Sets and Systems, International Journal of Approximate Reasoning, and International Journal of Computational Intelligence Systems, and furthermore, an Editorial Board member of journals Kybernetika, and Axioms. He also served as the Guest editors of several special issues of distinct journals. His research interests mainly include fuzzy modeling, especially fuzzy inference systems and fuzzy relational calculus. Research in this area led to the FUZZ-IEEE Best Paper Award in 2016 (Vancouver, Canada) for the paper “On the Satisfaction of Moser-Navara Axioms for Fuzzy Inference Systems”.

Generalizations of Choquet and Sugeno integrals for fusion of data with uncertainty

Javier Fernández

Public University of Navarra and Institute of Smart Cities, Spain

In recent years there has been an increasing interest on different extensions of Choquet and Sugeno integrals. These operators have shown themselves as very versatile tools to deal with many different types of problems, including some arising in fields such as classification, deep learning or decision making, among many others. In this talk, we will make a review of some recent theoretical developments in the study of generalizations of these operators. In particular, we will show how these generalized forms of Choquet and Sugeno integrals arise in a natural way from the classical fuzzy ones. However, these new operator display some interesting new properties, specially related to monotonicity. We will discuss some of the problems where they have shown its usefulness, with a particular focus on those related to deep learning and neuroscience.

Besides, it is well known that intervals can be used to handle uncertainty in information fusion processes. In this sense, we can understand the width of a given interval as a measure of the lack of certainty linked to a given data. But some operations, such as differences, are not well defined in general for closed non-empty subintervals of the unit interval, which implies that extending, fuzzy integrals, for instance, to the interval setting, may not be straightforward. We will discuss how we can overcome this difficulty and define generalizations of fuzzy integrals for intervals (and even more general settings), by means of the notion of d-integral. We will also see how we can tackle the problem of ordering the interval inputs when we are making use of these functions, and how they can be applied to some specific problems.

Biography

Javier Fernández is associate professor at the Department of Statistics, Computer Science and Mathematics of the Public University of Navarra since 2019. Member of the research group on Artificial Intelligence and Approximate Reasoning, his main research lines are focused on the problem of fusing information, mainly using fuzzy techniques, for applications such as image processing, decision making, classification, deep learning, data mining, machine learning or the computational brain. He has been the main researcher in a project of the National Research Plan and in two regional R&D projects and he has taken part in another 11 research projects with public funding. He has an h-index of 36 in Google Scholar, with 4819 citations. He has published around 80 papers in JCR journals, with 75% of them in Q1 journals. He is associated editor of the IEEE Transactions on Fuzzy Systems journal and co-editor of the Mathware&Soft Computing online magazine. He has been co-editor of two books and has co-advised three Ph.D. thesis, and he is now co-advising another two ones. He is research vicedirector of the new Navarra Artificial Intelligence International Center since 2023.

Fuzzy measures for metric learning and data-driven models

Vicenç Torra

Public university of Navarra and Institute of Smart Cities, Spain

Fuzzy measures are also known as non-additive measures, capacities and non-monotonic games. They generalize additive measures and probabilities by means of replacing the additive axiom by a monotonicity one. That is, given two sets A and B that are disjoint the measure of their union $\mu(A \cup B)$ can be smaller than, equal to, or greater than the addition of the measures $\mu(A) + \mu(B)$. In this way, we can express positive and negative interactions between elements. For example, $\mu(A \cup B) \geq \mu(A) + \mu(B)$ if there is a positive interaction. In this way, we can model situations that e.g. probabilities cannot satisfy. For example, for pairwise disjoint A, B and C we may have $\mu(A) < \mu(B)$ but in contrast $\mu(A \cup C) > \mu(B \cup C)$. This is not possible for probabilities as when $\mu(A) < \mu(B)$ we need also $\mu(A \cup C) < \mu(B \cup C)$.

Fuzzy measures can be used in combination with fuzzy integrals. Examples of fuzzy integrals are the Sugeno and the Choquet integral. The literature on fuzzy integrals is rich, each with its own properties and characteristics. The fuzzy integral of a function with respect to a fuzzy measure computes a kind of expected value, or an aggregated value, of the function. This is particularly evident in the case of the Choquet integral as when the measure is a probability, the Choquet integral of the function is exactly its expected value. If we have them defined on a discrete set, then, it is just the weighted mean.

In this talk we will discuss the use of fuzzy measures and integrals in applications related to machine learning. In particular, we will show how we can use them to define a distance that generalizes the (weighted) Euclidean distance. Then, we can learn the distance from examples. Therefore, we use these integrals to solve a problem of metric learning. We have applied this in data privacy to disclosure risk assessment. We will also discuss other applications of fuzzy measures and integrals related to building data-driven models.

References

Abril, D., Navarro-Arribas, G., Torra, V. (2012) Choquet integral for record linkage, *Ann. Oper. Res.* 195, 97-110.
Torra (2022) (Max, $\oplus$)-transforms and genetic algorithms for fuzzy measure identification, *Fuzzy Sets Syst.* 451, 253-265.
Türkarslan, E., Torra, V. (2022) Measure Identification for the Choquet Integral: A Python Module, *Int. J. Comput. Intell. Syst.* 15, 89.

Biography

Vicenç Torra is Professor on AI (Umeå University, Sweden). He is an IEEE and EurAI Fellow, and ISI elected member. He held positions at the Spanish Research Council (IIIA-CSIC), Maynooth University (Ireland), and Skövde University (Sweden). His fields of interests include privacy-preserving machine learning, and approximate reasoning (fuzzy sets, fuzzy measures, and aggregation functions).

He has written several books including "Modeling decisions" (with Y. Narukawa, Springer, 2007), "Data Privacy" (Springer, 2017), "Guide to Data Privacy" (Springer, 2022). He is founder and editor of the Transactions on Data Privacy. Since 2004 he is organizing the conference series Modeling Decisions for Artificial Intelligence.

Part II

EUSFLAT General Track

Fuzzy Conversational Character Computing

Sophie Hundertmark[1] and Edy Portmann[2]

[1] University of Applied Science Lucerne
[2] University of Fribourg

This abstract deals with the combination and complementation of the three existing research disciplines: Character Computing, Conversational Theory and Fuzzy Logic or Fuzzy Classification, so that the vision pursued by the researchers of Character Computing - computer systems that can autonomously adapt to the behaviour of their human interlocutors - is achieved. This primarily involves chat- and voicebots, also called conversational AI systems. These types of systems only rarely have access to sensor data. As a rule, they only exchange text or voice messages with the user.

Until now, researchers like El Bolock A. (2020) have determined the character of a user with fix values. They clearly delimit all characters from each other, and form fixed boundaries between the individual characteristics and the associated character models. However, there are often overlaps between the individual characters, the associated characteristic values, and the expected behavior of the user. The boundaries are therefore fuzzy. El Bolock A. et al. (2020) have already recognized this challenge and point out in their work that the application of fuzzy logic would be desirable in the future.

Thanks to the use of the multi-dimensionality described by Zumstein D. (2007), existing models of character computing or the previously fixed-defined characters can be "softened". Instead of fix boundaries between the individual characters and the behavior expected from them, fuzzy classification helps to determine the degrees of affiliation of users or their characteristic values and thus to form realistic characters.

To derive fuzzy classes and affiliations from sharp contexts or fix character boundaries, the attributes are regarded as linguistic variables and verbal terms are assigned to each equivalence class. The use of linguistic variables is already a common application in the discipline of Computing with Words and Perceptions. Thanks to the use of linguistic variables, i.e. words or word combinations, the equivalence classes of the attributes can be described more intuitively. Each term of the linguistic variables represents a fuzzy set. In the end, the computer system will not work with fix characters, but rather with "affiliations". Thus, the users then belong more or less to certain character groups and also their expectations are not always 100% unambiguous, but belong to certain groups.

Further, Zumstein D. (2007) notes that in comparison to sharp classifications, fuzzy classifications require fewer terms or equivalence classes to be defined in order to describe each attribute. This also seems to greatly simplify the development of the character models and still represent the characters closer to reality.

In addition to the reference to fuzzy logic, researchers in character computing also point out that there are limitations to the model calculation because the chat and voice bots lack important information from sensors. This challenge should be eliminated in the future by applying Conversational Theory. Pangaro (2010) argues in his research on Conversational Theory that conversations provide almost the only platform in which two social systems - including computers and humans - can exchange and adapt to each other. It should therefore be possible that the methods of Conversational Theory can supplement the missing information from sensors and that computer systems no longer need a multitude of sensor data as suggested by El Bolock et al. (2020). Instead, a cleverly guided conversation should help the computer system to determine the character of its counterpart and adapt its behavior accordingly. With each new message shared, the two counterparts get to know each other better, adapt to each other, build trust and eventually come to an agreement. Interestingly, Pangaro (2010) always writes of a joint agreement in which both interlocutors adapt to each other. In their research on character computing, however, El Bolock A. et al. (2020) only ever assume that the computer system adapts to the behaviour of the human user. This is probably another advantage of the combination of character computing and conversational theory. Now it is no longer only the computer that should adapt, the human also adapts and thus makes it easier for the computer to adapt. Apart from the fact that this mutual adaptation leads to the computer being able to adapt more quickly and easily, this approach also seems morally and ethically more correct. The imbalance between information gathering, processing and storage that often exists in classical character computing is much more balanced using Conversational Theory.

References:

Bolock, A.E., Abdelrahman, Y., & Abdennadher, S. (2020). Character Computing. Character Computing.
Pangaro, P. (2010). Rethinking design thinking. PICNIC Festival, Amsterdam, September.

Zumstein, Darius. (2007). Customer Performance Measurement - Analysis of the Benefit of a Fuzzy Classification Approach in Customer Relationship Management. 10.13140/RG.2.2.15682.58560.

Forecasting of streamflow for the Arga river passing through Pamplona some hours in advance

Ismael Moreno Lasa[1,*], Peio Oria Iriarte[2], and Humberto Bustince[1]

[1] Universidad Pública de Navarra, 2022 Campus de Arrosadía, 31006, Pamplona, Navarra
[2] Agencia Estatal de Meteorología (AEMET)
*ismaelmorenolasa@outlook.com, moreno.133550@e.unavarra.es

Keywords: Long Short-term Memory (LSTM) Neural Network · Recurrent Neural Network (RNN) · Streamflow forecast · Flood early warning

Modeling streamflow in fast-flowing watersheds is a highly complex problem where commonly used hydrological models often have limitations. [2] The existence of a prediction that allows for early warning of floods is vital for minimizing damage to property and infrastructure and reducing potential risks to people.

Machine learning techniques have the potential to overcome some of the limitations of traditional hydrological models by using large datasets to learn the relationships between different hydrological variables, making it possible to make more accurate predictions of streamflow in fast-flow basins. [4,6,5]

In this work, Long Short-term Memory (LSTM) neural networks are tested to predict streamflow in the Arga river's basin. LSTM networks are a type of Recurrent Neural Network (RNN) that are particularly well-suited for time-series prediction tasks. [8] These networks have memory cells that allow them to remember patterns in the data over a longer period of time, making them effective at capturing the temporal dependencies present in the streamflow data. The use of this types of networks allows the potential to overcome some of the typical limitations of traditional hydrological models. [1,3,7]

By using LSTM networks, we show that the model is able to capture the complex temporal dynamics of the streamflow data and make short-term, hours ahead, accurate predictions even for high flow scenarios. The results demonstrate that the use of LSTM networks for streamflow prediction in the Arga river basin is a promising approach, particularly for short-term, hours ahead, predictions.

The usage of machine learning approaches may unlock new potential in the forecasting and management of water resources in the area, as well as in risk assessment and early warning systems for floods.

Acknowledgments

The support of TESICNOR (Técnicas y Servicios de Ingeniería y Control del Norte S.L.) and Agencia Estatal de Meteorología (AEMET) to this research is appreciated, the contribution of TESICNOR and AEMET in providing the necessary resources and facilities has been crucial in the execution of the project.

References

1. Gohar, T., Hasan, L., Khan, G.M., Mubashir, M.: Constraint free early warning system for flood using multivariate lstm network. In: 2022 2nd International Conference on Artificial Intelligence (ICAI). pp. 64–70. IEEE (2022)
2. Kauffeldt, A., Wetterhall, F., Pappenberger, F., Salamon, P., Thielen, J.: Technical review of large-scale hydrological models for implementation in operational flood forecasting schemes on continental level. Environmental Modelling & Software **75**, 68–76 (2016)
3. Li, J., Qian, K., Liu, Y., Yan, W., Yang, X., Luo, G., Ma, X.: Lstm-based model for predicting inland river runoff in arid region: A case study on yarkant river, northwest china. Water **14**(11), 1745 (2022)
4. Mosavi, A., Ozturk, P., Chau, K.w.: Flood prediction using machine learning models: Literature review. Water **10**(11), 1536 (2018)
5. Parisouj, P., Mohebzadeh, H., Lee, T.: Employing machine learning algorithms for streamflow prediction: a case study of four river basins with different climatic zones in the united states. Water Resources Management **34**(13), 4113–4131 (2020)
6. Rasheed, Z., Aravamudan, A., Gorji Sefidmazgi, A., Anagnostopoulos, G.C., Nikolopoulos, E.I.: Advancing flood warning procedures in ungauged basins with machine learning. Journal of Hydrology **609**, 127736 (2022). https://doi.org/https://doi.org/10.1016/j.jhydrol.2022.127736, https://www.sciencedirect.com/science/article/pii/S0022169422003110

7. Yang, T., Sun, F., Gentine, P., Liu, W., Wang, H., Yin, J., Du, M., Liu, C.: Evaluation and machine learning improvement of global hydrological model-based flood simulations. Environmental Research Letters **14**(11), 114027 (2019)
8. Yu, Y., Si, X., Hu, C., Zhang, J.: A review of recurrent neural networks: Lstm cells and network architectures. Neural computation **31**(7), 1235–1270 (2019)

Systems of Fuzzy Relational Equations and Partially Defined Inputs

Nhung Cao and Martin Štěpnička

CE IT4I – IRAFM, University of Ostrava, Ostrava 701 03, Czech Republic nhung.cao@osu.cz, martin.stepnicka@osu.cz http://ifm.osu.cz

Keywords: Systems of fuzzy relational equations · Partial Fuzzy Set Theory · Modus ponens · Fuzzy rule-based systems Fuzzy rule-based systems need to be built correctly, which means that they should preserve some fundamental properties. The most typical one is the preservation of modus ponens, i.e., if the fuzzy input is equivalent to one of the antecedents, the derived output needs to be equivalent to the respective consequent. This leads to one of the classical topics of mathematical fuzzy modeling, in particular, systems of fuzzy relational equations. The most important question is whether such systems are solvable, i.e., whether there exists a fuzzy relation that models the given fuzzy rule base in such a way that the modus ponens is preserved.

There are plenty of known results in this area, however, the situation changes whenever we allow partial fuzzy sets to enter the systems. Here, the partiality is nothing else but the possibility to consider undefined membership degrees. Related (partial) 3-valued logics again belong to classical logical topics that are being studied since 1920's. Their extension to partial fuzzy logics was natural and it led to the partial fuzzy set theory as well. The application potential may be seen from the occurrence of missing values in databases, of various N/A values in questionnaires, and so on. The partial fuzzy set theory has been developed based on numerous extensions of partial algebras of truth values.

This background leads naturally to the fact that the (conditions and criteria of) solvability of partial fuzzy relational equations are strongly dependent on the chosen algebras. A recent publication uncovers the solvability questions and even the shape of the solutions for most of the known algebras however, it considers only the most general case. In particular, antecedents and consequents are partial fuzzy sets, and naturally, also the solution is a partial fuzzy relation. In this contribution, we revisit the problem and take into consideration a specific case that allows partiality only in the input. It means, that the (expert)knowledge stored in the antecedents, consequents, and consequently also the fuzzy relation modeling the fuzzy rule base, are fully defined. However, the input (observation) may be harmed by several undefined, e.g., missing, values, and we investigate what happens. We show, that under specific conditions, we still may preserve the modified modus ponens, i.e., that the inferred output is identical with the fully defined consequent of the respective rule.

Comparing Measures of Entropy in Interval-Valued Fuzzy Sets*

S. Cubillo, L. Magdalena, C. Torres-Blanc, J. Martínez-Mateo, and G. Sánchez-Torrubia

Universidad Politécnica de Madrid. Dpto. Matemática Aplicada.
28660 Boadilla del Monte, Madrid (Spain)
{scubillo,lmagdalena,ctorres,jmartinez,gsanchez}@fi.upm.es

Fuzzy entropy was defined by L.A. Zadeh in [1] for the first time, and in [2] A. De Luca and S. Termini established the required axioms to interpret the entropy as a measure of the amount of information. A. Kaufmann in [3] considered the entropy as the distance between a fuzzy set and its nearest crisp set, while R.R. Yager in [4] defined it as the distance between a fuzzy set and its complement. M. Zhang and Z. Wang in [5] return to the study of entropy in interval-valued fuzzy sets (fuzzy sets where the image of any element is a closed subinterval of $[0, 1]$), and provide a list of measures given by different authors throughout the literature. Among others, they focus on those given by P. Burillo and H. Bustince (we denote it by I_B) in [6], W. Zeng and H. Li (as E_Z) in [7], and I.K. Vlachos and G.D. Sergiadis (denoted by E_V) in [8]. Moreover, E. Szmidt and J. Kacpryk in [9] introduced an entropy measure for Atanassov intuitionistic fuzzy sets (AIFSs). It is well known that these sets are equipollent to interval-valued fuzzy sets (but not equal, as K. Atanassov and G. Gargov explained in [10]). So, we could translate that entropy, introduced for AIFSs, to interval-valued fuzzy sets obtaining a formula that we will denote by E_S.

A careful study of these measures shows that those authors consider the entropy from different points of view. The function of entropy I_B aims to establish how 'non-fuzzy set' an interval-valued set is, while E_Z, E_V and E_S focus on to calculate how 'non-crisp' it is. In such a way, for example, in the respective axiomatics, given an interval-valued fuzzy set A, $I_B(A) = 0$ if and only if A is a fuzzy set, and $E_S(A) = E_Z(A) = E_V(A) = 0$ if and only if A is a crisp set. As far as we know, no comparison has been made between these measures, and just this has been our goal in this work. Firstly, we have reinterpreted the formula of these measures by defining each interval in terms of its centroid and its half width. This has made the comparison easier. We then realized that, while the values of $E_Z(A)$ and $E_V(A)$ are not affected by the half width (they only refer to the centroid), $E_S(A)$ relates to both, the centroid and the half width of the interval assigned to each element. Furthermore, $I_B(A)$ is only influenced by the half width. Once this is done, it is not difficult to obtain some relationships between these measures. So, we have proved that E_V is smaller than or equal to E_S which, in turn, is smaller than or equal to E_Z. Finally, we have studied the relationship between the measure I_B and the rest of them. We have found out that I_B is smaller than or equal to E_S. Nevertheless, it is not comparable with E_V, that is, there exist interval-valued fuzzy sets A, such that $I_B(A)$ is smaller than $E_V(A)$, and there exist interval-valued fuzzy sets B, such that $E_V(B)$ is smaller than $I_B(B)$.

References

1. Zadeh, L.A.: Probability measures of fuzzy events. J. Math Anal. Appl. **23**, 421–427 (1968)
2. De Luca, A., Termini, S.: A Definition of a Nonprobabilistic entropy in the Setting of Fuzzy Sets Theory.Information and Control **20**, 301–312 (1972)
3. Kaufmann, A.: Introduction to the Theory of Fuzzy Sets. Academic Press, New York (1975)
4. Yager, R.R.: On measure on fuzziness and fuzzy complement. Internat. J. General Systems. **8** 169–180 (1982)
5. Zhang, M., Wang, Z.: Entropy and Semi-Entropies of regular Symmetrical Triangular Interval Type-2 Fuzzy Variables. Symmetry **14**(5), 930. https://doi.org/10.3390/sym14050930.
6. Burillo, P., Bustince, H.: Entropy on intuitionistic fuzzy sets and on interval-valued fuzzy sets. Fuzzy Sets and Systems. **78** 305–316 (1996)
7. Zeng, W., Li, H.: Relationship between similarity measure and entropy of interval valued fuzzy sets. Fuzzy Sets and Systems **157**, 1477–1484 (2006)
8. Vlachos. I.K., Sergiadis, G.D.: Subsethood, entropy and cardinality for interval-valued fuzzy sets - An algebraic derivation. Fuzzy Sets and Systems **158**, 1384–1396 (2007)
9. Szmidt, E., Kacprzyk, J.: Entropy for intuitionistic fuzzy sets. Fuzzy Sets and Systems **118**, 467–477 (2001)

* This research has been partially supported by the Government of Spain (grants PID2021-122905NB-C22 and PID2020-112502RB-C41), Comunidad de Madrid (Convenio Plurianual con la Universidad Politécnica de Madrid en la línea de actuación Programa de Excelencia para el Profesorado Universitario), and Universidad Politécnica de Madrid (Spain).

10. Atanassov, K., Gargov, G.: Interval valued intuitionistic fuzzy sets. Fuzzy Sets and Systems **31**, 343–349 (1989)

Time series aggregation in labelled fuzzy time series

L. Rodriguez-Benitez[1], J. Moreno-Garcia[1], E. Castillo-Herrera[1], J. Liu[2], L. Jimenez-Linares[1]

[1] Universidad de Castilla-La Mancha
[2] University of Ulster
luis.rodriguez@uclm.es, juan.moreno@uclm.es, ester.castillo@uclm.es, j.liu@ulster.ac.uk, luis.jimenez@uclm.es

A Time Series (TS) can be defined as a set of regular observations ordered in time of a quantitative characteristic from a phenomenon measured at consecutive points in time. It is possible to use fuzzy logic to model time series and this allows to approximate functions incorporating the readability associated with linguistic variables. One option for modelling these series is the use of Fuzzy Time Series (FTS) which are composed of fuzzy sets [1]. In this paper we propose a way to represent TS sets by means of FTS composed by linguistic labels previously defined in a set. The process consists of two steps. The first one obtains a FTS from the input TS sets using the method presented in [2]. In the second one the obtained FTS is transformed into a FTS composed of linguistic labels that have been selected from a predefined set of labels defined over the application domain. To perform the correspondence between each fuzzy set and linguistic label, the percentage of overlap between them is estimated using an approach very similar to the Monte Carlo integral [3]. The presented approach has been used for the generation of linguistic descriptions of noise pollution data.

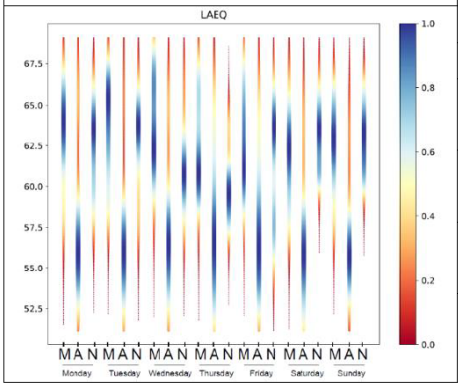
Picture	Time period	FTS Label	Label
	Monday M	H <=	(**H:0.921**) (M:0.176)
	Monday A	L <=	(**L:0.952**) (M:0.147)
	Monday N	H or M <=	(**H:0.615**) (**M:0.472**)
	Thursday M	H <=	(**H:1.000**) (M:0.326) (VH:0.013)
	Thursday A	L <=	(**L:0.974**) (M:0.134) (VL:0.003)
	Thursday N	H <=	(**H:0.616**) (M:0.102)

Table 1. An example of FTS output obtained. First column: a color code is used, where darker colors represent higher membership grade. Second column: M, A and N indicates Morning, Afternoon and Night. Third column: The set of labels used is {Very Low, Low, Medium, High, Very High} represented as {VL,L,M,H,VH}. Fourth column: The percentage of overlap that allows the label to be selected is indicated.

Table 1 shows an example of use in a concrete example. The labels that have an overlap percentage higher (indicated in bold) than a specific value, 0.45 in the example, are selected. As future work, the presented method will be used for the generation of linguistic descriptions of large sets of time series. The new method will have to be adapted and formalized to be able to deal with these large sets.

References

1. Q. Song, B.S. Chissom, Fuzzy time series and its models, Fuzzy Sets and Systems, vol. 54(3), 1993.
2. L. Rodriguez-Benitez, J. Moreno-Garcia, E. Castillo-Herrera, J. Liu, L. Jimenez-Linares, Aggregation and Definition of an Algebraic Framework over Fuzzy Time Series: an Application in the Supply-Demand Domain, pg. 104-115, Vol. 149, 2022.
3. C. P. Robert, G. Casella, Montecarlo integration. In Monte Carlo statistical methods, pp. 71-138, Springer, New York, NY, 1999.
4. J. Moreno-Garcia, L. Jimenez-Linares, J. Liu, L. Rodriguez-Benitez, Generation of linguistic descriptions for daily noise pollution in urban areas, IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), 1-6, 2021.

Python library for interval-valued fuzzy inference

Krzysztof Dyczkowski[1], Barbara Pękala[2,3], Piotr Grochowalski[2], Dawid Kosior[2], Dorota Gil[3], and Wojciech Koziol[2]

[1] Adam Mickiewicz University, Poznań, Poland chris@amu.edu.pl
[2] University of Rzezszów, Rzeszów, Poland {bpekala,pgrochowalski,dkosior,wkoziol}@ur.edu.pl
[3] University of Information Technology and Management, Rzeszów, Poland dgil@esiz.edu.pl

Uncertainty, which is an indispensable part of many concepts, data, and operations present in various applications related to computer science and engineering or medical diagnostics, requires an approach that takes it into account. The theory of interval-valued fuzzy sets, in particular, the interval model of inference (generalized fuzzy inference), proved to be useful for handling uncertainty. In this presentation, we propose Interval-Valued Inference System (IFIS) based on Simpful package, a universal and user-friendly Python library designed for: defining, analyzing, and interpreting fuzzy inference systems. The extended library provides lightweight application programming. The interface allows you to intuitively define interval fuzzy sets and interval rules and to perform generalized fuzzy inference. We provide some practical examples showing that our library extension is a valuable addition to open-source software that supports generalized fuzzy inference.

Criticism of the center of gravity defuzzification

Michal Burda[1], Mirko Navara[2], and Martin Štěpnička[1]

[1] CE IT4I – IRAFM, University of Ostrava, Ostrava 701 03, Czech Republic michal.burda@osu.cz, martin.stepnicka@osu.cz http://ifm.osu.eu
[2] Department of Cybernetics, Faculty of Electrical Engineering, Czech Technical University in Prague, Prague 121 35, Czech Republic
navara@fel.cvut.cz, http://cmp.felk.cvut.cz/~navara

Keywords: Defuzzification · Center of gravity · Steiner centroid · Fuzzy system

The center of gravity is one of the most popular methods of defuzzification in fuzzy systems. It seems natural to use the horizontal position of the center of gravity of the area under the membership function as a representative of the (horizontal) position of the fuzzy set. The more the membership function at some point, the higher its influence on the result. However, we dare to point out some drawbacks of this choice and suggest also some alternatives.

The values with the highest membership degree seem to be emphasized when we look at the vertical representation. It does not seem so if we consider the horizontal representation (by cuts/level sets of the described fuzzy set). Each of the cuts contributes proportionally to its **width**, which represents the **imprecision** of the localization. The highest cuts are the **shortest**, thus they have the **least** contribution to the result of defuzzification, although they are **narrow**, which means that they express the localization of the fuzzy set **most precisely**. From this point of view, it seems that the best information receives very low weight in the aggregation. In the extreme case, the 1-cut (=the core) may be a singleton, suggesting that this is the best possible value and a desirable result of defuzzification, but it receives a zero weight in the information fusion (provided that some lower cuts are of non-zero measure, hence the area under the membership function is non-zero).

Looking for an alternative compensating the above-mentioned drawback, we may suggest, e.g., the Steiner centroid (also called Steiner point, see the references for details), which was generalized from convex crisp sets to convex fuzzy sets in [3]. Originally, a Steiner centroid of a **convex crisp** set in vector space $\mathbb{R}^n$ was defined in [1]. As proved in [2], it is the only function s from non-empty compact convex sets to points with the following properties:

1. $s(A+B) = s(A) + s(B)$,
2. $s(t(A)) = t(s(A))$ for any rigid motion t (i.e., t is a combination of a rotation and symmetry; an isometry),
3. s is continuous (w.r.t. the Hausdorff metric on the collection of non-empty compact convex sets).

For a bounded non-convex set, the Steiner centroid is the same as for its closed convex hull. This reduces its robustness, and that might be the reason why it was not used more often. In the special case of a one-dimensional space, a bounded convex **crisp** set is an interval and its Steiner centroid is just its center, which coincides with the center of gravity.

In [3], the Steiner centroid was generalized to (convex) fuzzy sets by requiring the same properties (1)–(3). It was proved that these conditions are satisfied for a large family of mappings. They all compute the Steiner centroids of all cuts and then aggregate them to a single value by integrating w.r.t. a variable u over the set of all membership degrees, i.e., the interval $[0,1]$. The integrand can be multiplied by almost any weighting factor $w(u)$, which depends on u, but not on the argument (fuzzy set). The choice of weighting function w gives the ambiguity of the Steiner centroid for fuzzy sets. In any case, it requires to defuzzify each u-cut separately (in the special case of one dimension, just use the arithmetic mean of the minimum and maximum of the u-cut) and then integrate these values w.r.t. u. The weighting function w can be chosen constant, then all cuts contribute equally to the result of defuzzification. We can also assign higher weights to higher cuts, expressing their "higher precision". When applied to bounded convex **fuzzy** sets, the results obtained by Steiner centroid differ from the center of gravity even in one dimension. For symmetric fuzzy sets, both approaches coincide, giving the center of symmetry as the result.

We think that Steiner centroids offer a promising alternative which can mitigate the disadvantage of the center of gravity. We can imagine also other alternatives. In any case, we want to attract attention to them, instead of using the center of gravity without hesitation.

There is another problem in defuzzification: values at the end of the domain. Usually the least and the greatest linguistic value is represented by a fuzzy set whose membership function does not go to zero continuously, but remains maximal till the end of the domain of possible values. Then the result of defuzzification depends in the choice of the bounds of the domain, which are not always obvious. Further, the center of gravity gives necessarily a result distant from the margin (and many other defuzzification methods do so, too). The proposed defuzzification using Steiner centroids also suffers this drawback, but it does not manifest so severely. We dare to conclude that the center of gravity should not be the first choice among defuzzification methods and that the Steiner centroid or other alternatives should be considered.

Acknowledgements: The second author was supported by the Czech Science Foundation grant 19-09967S.

References

1. Schneider, R.: On Steiner points of convex bodies. Isr. J. of Math. **9**, 241–249 (1971)
2. Shephard, G.: A uniqueness theorem for the Steiner point of a convex region. Journal of the London Mathematical Society **43**, 439–444 (1968)
3. Vetterlein, T., Navara, M.: Defuzzification using Steiner points. Fuzzy Sets and Systems **157**, 1455—–1462 (2006)

Detecting Radical Profiles on Social Media: A Fuzzy-Based Approach for Homeland Security

Andres Montoro, Jared D.T. Guerrero-Sosa, Jose A. Olivas, Francisco P. Romero, and Jesus Serrano-Guerrero

Dpt. Inf. Systems and Technologies, University of Castilla La Mancha, Paseo de la Universidad, 4, Ciudad Real (Spain)
{andres.montoro,jareddavidtadeo.guerrero}@alu.uclm.es
{joseangel.olivas,franciscop.romero, jesus.serrano}@uclm.es

While offering global communication with speed and anonymity, social networks also have a dark side, as they can be exploited for terrorism-related activities. Such malicious uses can be broadly classified into categories like propaganda, financing, training, planning and executing terrorist acts, and cyberattacks. Several works in the literature study the radical phenomenon in social media, from its detection [2], through prevention [1], to reactions to different terrorist attacks [3].

A hybrid model for detecting radicals in social media is proposed, employing a fuzzy taxonomy to represent radical behaviour and a fuzzy ontology to decrease the abstraction level for radical detection within a specific domain.

- Characterization of the radical profile: Social networks have become a new breeding ground for immersion in the radicalisation process that did not exist before. To identify and understand radical behaviour in social media, our approach focuses on analyzing and proposing methods for knowledge acquisition, representation and utilization. We have used a hybrid method of knowledge acquisition to construct the taxonomy to characterize the radical profile in a social media environment by analyzing reference literature on radical behavior [5,6,7] and resulting three components:
  - Behavioral features: conducts that incite and/or legitimize radical behaviour.
  - Personal features: refer to how the individual is immersed in the radicalization process, emphasizing social and individual characteristics and their desires and wishes.
  - Environmental features: refer to the climate favouring radicalisation and the transition to the terrorist act.
- Fuzzy Inference Approach: The knowledge base consists on term sets used to describe each of the taxonomy classes and fuzzy numbers (normalized, continuous and convex fuzzy sets) that define the semantics within a predefined domain of each of the taxonomy labels and a rule base composed from the linguistic terms defined in the database. It follows the multiple inputs-single output form. The inference engine uses the extension of the classical Modus Ponens, called *Generalized Modus Ponens* proposed by Zadeh in [4].

The model is deployed in the Phoenix Platform from Mollitiam Industries. Mollitiam Industries is a European company specializing in designing and developing modular software for cyber intelligence with applications in Homeland Security (HLS) and Homeland Defense (HLD). Phoenix cyberintelligence digital platform is a modular massive monitoring system to produce intelligence from the anonymous download of unstructured data from Social Networks, Darknets and Deepwebs. The product aims to detect radicals in social media using a taxonomy that models radical behaviour and an ontology that allows the detection of specific domains within radical behaviour. To exemplify this, it is reduced to a prototype that models only the personal and environmental features of the taxonomy and a specific domain is selected for the construction of the ontology (jihadism, religious extremism). The prototype is easily scalable to different domains. Adding more metadata and extending to the whole taxonomy will allow assessing an individual's radicalisation level. Since the model is considered a prototype and a first approximation to the final product, some limitations are evident. Now the process only works with text, but since the ontology models concepts, it is scalable to work with images or audio since these formats have transcription mechanisms.

Acknowledgments

This work has been partially supported by FEDER and the State Research Agency (AEI) of the Spanish Ministry of Economy and Competition under grant SAFER: PID2019-104735RB-C42 (AEI/FEDER, UE).

References

1. Burnap P., Rana O., Avis N., Williams M. L., Housley W., Edwards A., Morgan J., and Sloan L.: Detecting tension in online communities with computational Twitter analysis. Technological Forecasting and Social Change **95**, 68–73 (2015).
2. Nouh M., Nurse J. R. C., and Goldsmith M.: Understanding the radical mind: Identifying signals to detect extremist content on Twitter. 2019 IEEE International Conference on Intelligence and Security Informatics (ISI), 98–103 (2019).
3. Burnap P., Williams M. L., Sloan L., Rana O., Housley W., Edwards A. et al.: Tweeting the terror: modelling the social media reaction to the Woolwich terrorist attack. Social Network Analysis and Mining **4**(1), 1–14 (2014).
4. Zadeh L. A.: Outline of a New Approach to the Analysis of Complex Systems and Decision Processes. IEEE Transactions on Systems, Man and Cyb. **SMC-3** (1), 28–44 (1973).
5. Borum R.: The Etiology of Radicalization, in The Handbook of the Criminology of Terrorism. John Wiley & Sons, 17–32 (2017).
6. Webber D., and Kruglanski A. W.: Psychological Factors in Radicalization: A "3 N" Approach, in The Handbook of the Criminology of Terrorism. John Wiley & Sons, 33–46 (2017).
7. Sawyer J. P., and Hienz J.: What Makes Them Do It? Individual-Level Indicators of Extremist Outcomes, in The Handbook of the Criminology of Terrorism. John Wiley & Sons, 47–61 (2017).

A shared vision of similarity and inclusion measures for IVFSs

Mohammad Ojaghi, Agustina Bouchet, Irene Mariñas-Collado, and Susana Montes

Department of Statistics and Operational Research, University of Oviedo, Spain
{ojaghimohammad.uo,bouchetagustina,marinasirene,montes}@uniovi.es
https://unimode.grupos.uniovi.es

Keywords: interval-valued fuzzy sets · measures of order degree between intervals · inclusion measures · similarities.

This work considers interval-valued fuzzy sets (IVFSs) from an epistemic perspective [5,6,7,8,11,4]. Thus, we assume that there is one actual, real-valued membership degree of an element inside the membership interval of possible membership degrees. This makes the intervals an essential tool for IVFSs. Thus, for example, their width directly affects the lack of information about the considered IVFS.

Starting from this premise, measures of order degree between intervals are considered (denoted by o) [1,4]. With these measures, inclusion measures for IVFSs on finite referentials (I) [3,3] can be generated by simply combining the comparison made between the two intervals that define both sets at each point of the referential through aggregation measures (in particular, the arithmetic mean). In this procedure, it is evident that an essential step will be the order between intervals considered to compare them and, therefore, define the inclusion between IVFSs.

On the other hand, a measure of order degree between intervals (o) can also be used to measure the degree of similarity between two intervals (s), simply by combining the degree to which each interval is smaller than the other through an appropriate aggregation measure (increasing and one-strict) [1].

Finally, it is evident that by combining one by one, for the elements of a finite referential, the similarity between the intervals (s) that define the degree of membership at each point, a decomposable similarity measure for IVFSs could be obtained (S) [9,5].

The main purpose of this work is to present all these concepts and relate them, as well as to analyze the degree of compatibility between the similarity measure for IVFSs defined from the inclusion measure obtained from the measure of order degree between intervals (IS) and the measure obtained point by point from the similarity between the respective intervals (S). In summary, we have $o \to I \to IS$ and $o \to s \to S$ and we compare IS and S, that is, we compare two different ways to generate similarity measures for IVFSs starting from measures of order degree between intervals.

Acknowledgements

M. Ojaghi was supported by Erasmus+ KA131 Program.

References

1. G. Beliakov, A. Pradera, T.: Calvo. Aggregation Functions: A Guide for Practitioners. Springer, 2007.
2. A.Bouchet, M. Sesma, H. Bustince, I. Díaz, S. Montes.: Measures of embedding for interval-valued fuzzy sets. Fuzzy Sets and Systems, doi: https://doi.org/10.1016/j.fss.2023.03.008.
3. H. Bustince.: Indicator of inclusion grade for interval-valued fuzzy sets. Application to approximate reasoning based on interval-valued fuzzy sets. International Journal of Approximate Reasoning. 23(3) (2000) 137–209.
4. H. Bustince, J. Fernandez, A. Kolesárová, R. Mesiar.: Generation of linear orders for intervals by means of aggregation functions. Fuzzy Sets and Systems. 220 (2013) 69–77.
5. H. Bustince, E. Barrenechea, M. Pagola, J. Fernández, Z. Xu, B. Bedregal, J. Montero, H. Hagras, F. Herrera, B. De Baets.: A Historical Account of Types of Fuzzy Sets and Their Relationships. IEEE Transactions on Fuzzy Systems. 24(1) (2016) 179–194.
6. I. Couso, D. Dubois.: Statistical reasoning with set-valued information: Ontic vs. epistemic views. International Journal of Approximate Reasoning, 55(7) (2014) 1502–1518.
7. I. Grattan-Guiness.: Fuzzy membership mapped onto interval and many-valued quantities. Zeitschrift für mathematische Logik und Grundlagen der Mathematik, 22(1) (1976) 149–160.
8. K.-U. Jahn.: Intervall-wertige mengen. Mathematische Nachrichten, 68(1) (1975) 115–132.
9. S. Kabir, C. Wagner, T. C. Havens and D. T. Anderson: A Similarity Measure Based on Bidirectional Subsethood for Intervals. IEEE Transactions on Fuzzy Systems, 28/11 (2020) 2890–2904.

10. B. Pekala, K. Dyczkowski, P. Grzegorzewski, U. Bentkowska.: Inclusion and similarity measures for interval-valued fuzzy sets based on aggregation and uncertainty assessment. Information Sciences, 547 (2021) 1182–1200.
11. R. Sambuc.: Fonctions ϕ-floues. Application l'aide au diagnostic en pathologie thyroidienne, Ph.D. Thesis, Université Marseille, Marseille, 1975.
12. L.A. Zadeh.: The concept of a linguistic variable and its application to approximate reasoning-I. Information Sciences, 8(3) (1975) 199–249.
13. W. Zeng, P. Guo.: Normalized distance, similarity measure, inclusion measure and entropy of interval-valued fuzzy sets and their relationship. Information Sciences, 178 (2008) 1224–1342.

Explainable crowd decision making methodology

Cristina Zuheros[1], Eugenio Martínez-Cámara[1], Enrique Herrera-Viedma[1], Iyad A. Katib[2], and Francisco Herrera[1]

[1] Andalusian Research Institute in Data Science and Computational Intelligence (DaSCI), University of Granada, 18071, Granada, Spain
czuheros@ugr.es, {emcamara,viedma,herrera}@decsai.ugr.es
[2] Faculty of Computing and Information Technology, King Abdulaziz University, Jeddah 21589, Saudi Arabia
iakatib@kau.edu.sa

Keywords: crowd decision making · explainability · attention mechanisms · subgroup discovery · social media

Explainability is defined as an interface between humans and a decision maker which is comprehensible to humans and accurate to the decision maker [1]. It is convenient to incorporate explainability into decision making models, which generates a ranking of alternatives to solve a decision situation, so that people can further trust them and they can be further used at practical level. A relevant environment to acquire opinions for developing decision making models without the need of hunting for expert evaluations are social network platforms. These media collect the opinions of billions of people around the world, so they capture the wisdom of the crowd [2]. They offer unconstrained natural language evaluations which are suitable to design explainable decision making models since such evaluations are already easy comprehensible to humans.

We propose an explainable crowd decision making methodology that automatically reveal the relevant information of its internal decision process and captures the wisdom of crowds available on social media. Specifically, it is an a posteriori crowd decision making system since it incorporates a backward mechanism which generates easily understandable explanations in natural language that indicate why an alternative is chosen as the best. The proposal is based on two key elements. On the one hand, it incorporates attention mechanisms into a novel sentiment analysis method to discover the relevant sentences of the natural language expert evaluations for the achieved ranking. On the other hand, it considers subgroup discovery algorithms to unveil the relevant aspect terms related to the criteria of the alternatives for the decision reported

The workflow of the proposed methodology is depicted in three steps:

1. The expert opinions are extracted from their plain text evaluations using the sentiment analysis method, which is a multitasking neural network that infers the aspect terms, category, and polarity components of the opinions. The categories match the criteria of the alternatives. Then, we represent the opinions of all the experts that refer a particular alternative and criterion into a table that we named bag of opinions by criteria or BOC.
2. The crowd decision making process is conducted. We aggregate the expert opinions associated to each alternative and criterion mainly taking into account the frequency of positive opinions over the total opinions from the BOC tables. Then, an exploitation stage obtains a ranking of the alternatives through a weighted average of the collective evaluation.
3. The explainable backward process is performed to uncover the meaningful information that justify the achieved ranking. It identifies the criteria, the aspects terms, and the sentences with higher influence for the reached solution by analyzing the collective evaluation, by applying subgroup discovery in the BOC tables, and by examining the attention weights from the sentiment analysis method, respectively.

Acknowledgements This work was partly supported by the grants PID2020-119478GB-I00, PID2019-103880RB-I00 and PID2020-116118GA-I00 funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe". C. Zuheros is supported by the grant PRE2018-083884 funded by MCIN/AEI/10.13039/501100011033 and by "ESF Investing in your future".

References

1. Guidotti, R., Monreale, A., Ruggieri, S., Turini, F., Giannotti, F., & Pedreschi, D. (2018). A survey of methods for explaining black box models. ACM computing surveys (CSUR), 51(5), 1-42.

2. Zuheros, C., Martínez-Cámara, E., Herrera-Viedma, E., & Herrera, F. (2022). Crowd Decision Making: Sparse Representation Guided by Sentiment Analysis for Leveraging the Wisdom of the Crowd. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 53(1), 369-379.

Comparison between Fuzzy and Neuro-Fuzzy Inference Systems in Cloud Computing Scheduling

Francisco Javier Maldonado Carrascosa[1], Antonio Jiménez Sánchez[1], Sebastián García Galán[1], José Enrique Muñoz Expósito[1], Doraid Seddiki[1], and Adam Marchewka[2]

[1] Universidad de Jaén, Spain
[2] Politechnika Bydgoska, Poland
{fjmaldon, anjimene, sgalan, jemunoz}@springer.com, ds000025@red.ujaen.es, adam.marchewka@pbs.edu.pl

Nowadays, cloud computing services become popular due to their flexible functioning in multiple fields [1]. This kind of services operates on data centers that contain several computational equipment, storage, and communication networks offering a clearly improvable performance in terms of total execution time. In order to mitigate this problem, some studies have tried to reduce the amount of workload time that a data center has to manage [2]. In this regard, the workload can be allocated among different data centers to minimize the execution time through an efficient scheduling [3].

In this context, the use of Artificial Intelligence (AI) in cloud computing can improve cloud performance and efficiency. Thus, multiple AI techniques have been studied in several fields for achieving that goal such as genetic fuzzy systems (GFS) [4] or those based on particle swarm optimization (PSO) [5].

In this work, knowledge acquisition techniques based on fuzzy rule-based systems have been addressed. Moreover, a comparison between two different systems has been considered. The first methodology is based on Knowledge Acquisition with a Swarm Intelligence Approach (KASIA) [6], which consists of knowledge acquisition using PSO optimization considering a fuzzy inference system (FIS) that makes use of Mamdani if-then rules. In addition, this paper introduces another methodology based on an Adaptive Neuro-Fuzzy Inference System (ANFIS) [7], which consists of a FIS that is carried out by means of adaptative networks and makes use of Sugeno if-then rules. So, this paper studies the comparison between KASIA and ANFIS considering different cloud computing scenarios regarding task scheduling in order to minimize the workload execution time. To be precise, the studied system is composed of a meta-scheduler that distributes tasks among data centers in an efficient way.

For proper management of data, both systems are simulated considering several scenarios with different complexity in a simulator combining WorkflowSim and CloudSim features [8]. Additionally, this cloud simulator makes use of Pegasus workflow structures such as Montage or CyberShake [9] to recreate a cloud system with real workflow traces, which have been considered in this work. The configuration of the proposed systems is critical in the simulation stage. In this sense, five rules consisting of five antecedents and one consequent have been considered for the rule base (RB) in both methodologies. The antecedents are the MIPS, the IDLE consumed power (pow), the task's size (len), the maximum consumed power (pmax), and the CPU utilization (use), while the output is the data center selection (sel) for a virtual machine. Three membership functions have been considered for the five inputs (low, middle and high), while five have been established for the consequent (very low, low, middle, high, very high).

For the KASIA technique, a population of 20 particles and 100 iterations have been studied for having a low computational cost setting. Besides, three more parameters are necessary for KASIA's rule discovery: the initial particle inertia which is 0.9, and the social and individual factors set as 2. On the other hand, the ANFIS technique considers one additional parameter besides the RB's: the epoch variable which is the number of times the RB is trained, in this case, 50 epochs have been contemplated. It has to be accentuated that the ANFIS configuration is set with fewer variables than KASIA one. Moreover, some scenarios with a network topology of 20 virtual hosts, and 20 virtual machines have been analysed since this is the standard configuration of the simulator. The output of the simulator is the elapsed simulation time.

Once the configurations have been set, the workflows have to be stated. In this sense, work traces from the Montage and CyberShake projects with 100 jobs have been processed for evaluating the meta-schedulers. For every simulation, the makespan variable is considered as fitness for obtaining a more accurate configuration and to see its final convergence value where experiments correspond to the best solution average of 30 generations at every iteration. For ensuring the well-functioning of the learning process, both methodologies are validated with good RBs. Results prove that ANFIS slightly outperforms KASIA (0,3%) in terms of total execution time (makespan) when using settings in KASIA with low computational cost. Furthermore, ANFIS improves KASIA results by 7.01% and 6.33% in the Montage and CyberShake scenarios, respectively,

in terms of computational cost in time. Therefore, the ANFIS meta-scheduler is also better in terms of the number of RB evaluations, surpassing the KASIA technique by 60% in this sense.

Acknowledgements This work has been supported by the research projects P18-RT-4046 and PID2020-119082RB-C21, funded by the Andalusian and Spanish Governments, respectively, and by the NextGenerationEU recovery plan, funded by the European Union.

References

1. A. Abidi et al. Combining 2D encoding and convolutional neural network to enhance land cover mapping from Satellite Image Time Series. Engineering Applications of Artificial Intelligence, vol. 122, 2023. https://doi.org/10.1016/j.engappai.2023.106152
2. T. Daradkeh et al. Cloud Workload and Data Center Analytical Modeling and Optimization Using Deep Machine Learning. Network, Multidisciplinary Digital Publishing Institute, vol. 2, no. 4, pp. 643-669, 2022. https://doi.org/10.3390/network2040037
3. T. Khan et al. Workload forecasting and energy state estimation in cloud data centres: ML-centric approach. Future Generation Computer Systems, vol. 128, pp. 320-332, 2022. https://doi.org/10.1016/j.future.2021.10.019
4. O. Cordón et al. Genetic Fuzzy Systems: Evolutionary Tuning and Learning of Fuzzy Knowledge Bases. World Scientific, Advances in Fuzzy Systems – Applications and Theory, vol. 19, pp.488, 2001. https://doi.org/10.1142/4177
5. S. García Galán et al. Swarm Fuzzy Systems: Knowledge Acquisition in Fuzzy Systems and Its Applications in Grid Computing. IEEE Transactions on Knowledge and Data Engineering, vol. 26, no. 7, pp. 1791-1904, 2014. https://doi.org/10.1109/TKDE.2013.118
6. R. J. Pérez de Prado et al. Knowledge Acquisition in Fuzzy Rule-Based Systems with Particle Swarm Optimization. IEEE Transactions on Fuzzy Systems, vol. 18, no. 6, pp. 1083-1097, 2010. https://doi.org/10.1109/TFUZZ.2010.2062525
7. J. R. Jang. Fuzzy modeling using generalized neural networks and Kalman filter algorithm. Proceedings of the 9th National Conference on Artificial Intelligence, vol. 2, pp. 762-767, 1991. https://doi.org/10.5555/1865756.1865795
8. I.T. Cotes Ruiz et al. Dynamic Voltage Frequency Scaling Simulator for Real Workflows Energy-Aware Management in Green Cloud Computing. Plos One, vol. 12, no. 1, 2017. https://doi.org/10.1371/journal.pone.0169803
9. E. Deelman et al. The Evolution of the Pegasus Workflow Management Software. Computing in Science Engineering, 21(4), 22–36, 2019. https://doi.org/10.1109/MCSE.2019.2919690

Optimized interpretability for Expert Virtual Machine Migrations among Data Centers using Fingrams

Doraid Seddiki[1], Antonio Jiménez Sánchez[1], Francisco Javier Maldonado Carrascosa[1], Sebastián García Galán[1], José Enrique Muñoz Expósito[1], and Tomasz Marciniak[2]

[1] Universidad de Jaén, Spain
[2] Politechnika Bydgoska, Poland
ds000025@red.ujaen.es, {anjimene,fjmaldon,sgalan,jemunoz}@ujaen.es, Tomasz.Marciniak@pbs.edu.pl

Keywords: Expert systems · GUAJE · Fingrams · Interpretability · Cloud Computing · Virtual machine migration · Follows the renewable.

Cloud data centers (CDC) integrate a large number of computers, storage and communication networks to provide flexible and robust on demand cloud computing services [1]. The continuous operation of CDC implies a high electrical energy consumption [2]. Therefore, many of these centers use renewable energy to cover part of their power supply needs and, in turn, this use of renewable energy allows modular data centers to move the workload among different nodes based on renewable energy availability using load migration techniques.

Artificial intelligence can improve performance and efficiency in cloud computing. This work shows the optimization of the interpretability of a Rule Base (RB) taking advantage of a framework founded on CloudSim with Virtual Machine (VM) migration capabilities based on an expert system [1][3] and the GUAJE software tool [4]

The framework is used to create an input test data file for GUAJE and to obtain the different simulation results. It implements a Mamdani type [5] fuzzy rule-based system (FRBS) as algorithm for the meta-scheduler. The input variables are Cloud Data Center Renewable Availability (CDC-RA); Host Computational Capacity (HCC); Host Computational Availability (HCA); VM Maximum Computational Needs (VM-MCN); and VM Current Computational Needs (VM-CNN). The output is the suitability of a host to receive a VM that migrates. Each featured is modeled with 3 Gaussian type fuzzy membership sets (low, medium and high) except the output which is modeled with 5 sets (very not suitable, not suitable, suitable, very suitable and extremely suitable).

The rules of the meta-scheduler were created by the authors and are presented below:
If HCC is high and VM-MCN is low then OUTPUT is extremely suitable
If HCC is low and VM-MCN is high then OUTPUT is very not suitable
If HCA is high and VM-CNN is low then OUTPUT is extremely suitable
If HCA is low and VM-CNN is high then OUTPUT is very not suitable
If CDC-RA is low then OUTPUT is very not suitable
If CDC-RA is medium then OUTPUT is suitable
If CDC-RA is high then OUTPUT is very extremely suitable

On the other hand, GUAJE is used for the generation of fingrams, networks that graphically represent the interaction among rules, and the calculation of other quality parameters about the RB such as Interpretability Index (II) [6], coverage or accuracy in order to optimize the interpretability of the RB.

To test the behavior of the RBs, different simulations have been carried out in 3 scenarios.The first scenario is composed of 350 VMs, 265 hosts and 1500 cloudlets; the second scenario is composed of 695 VMs, 530 hosts and 5000 cloudlets; finally, the third scenario consist of 1052 VMs, 800 hosts and 10000 cloudlets.

The optimized RB obtained with this methods is as follows:
If HCC is high and VM-MCN is low then OUTPUT is extremely suitable
If HCA is high and VM-CNN is low then OUTPUT is extremely suitable
If CDC-RA is low then OUTPUT is very not suitable
If CDC-RA is medium then OUTPUT is suitable
If CDC-RA is high then OUTPUT is very extremely suitable

Experimental results show that the optimized RB provides similar or improved simulation results. Using the system defined in [6] the original RB obtained an index of 0.6 and the optimized one got a value of 0.6397, which supposes an increment of 6.62%. Using the II calculation system from GUAJE tool, the original RB obtained a value of 0.631 while the optimized one got an II of 0.777, resulting in an increase of

23.14%. For all scenarios total energy and renewable energy consumption remains similar, but for execution time there are significant changes. Regarding the first scenario, the execution time improves by 1.2%. For the second scenario, the execution time is reduced by 1.6%. For the third scenario, the execution time improves by 1.83%. As a conclusion, improvements in the performance of cloud data centers can be obtained while improving the interpretability of the corresponding FRBS.

Acknowledgements This work has been supported by the research projects P18-RT-4046 and PID2020-119082RB-C21, funded by the Andalusian Government and Spanish Government respectively.

References

1. D. Seddiki, S. Galán, J. E. Expósito, M. valverde, T. Marciniak, R. J. Pérez, "Sustainability-based Framework for Virtual Machines Migration Among Cloud Data Centers", 15th International Conference on Signal Processing and Communications Systems (ICSPCSÇ), 2021.
2. Meijer G. Cooling energy-hungry data centers. Science 2010;328(5976):318-9.
3. D. Seddiki, S. Galán, J. E. Expósito, M. valverde, T. Marciniak, R. J. Pérez, "Sustainable expert virtualmachine migration in dynamic clouds", Computers and Electrical Engineering 102 (2022) 108257.
4. D. P. Pancho, J. M. Alonso, L.Magdalena, "Quest for Interpretability-Accuracy Trade-off Supported by Fingrams into the Fuzzy Modeling Tool Guaje", International Journal of Computational Intelligence Systems, June 2013.
5. E. H. Mamdani, "Application of Fuzzy Logic to approximate reasoning using linguistic systems," IEEE Transactions on Computers, 26(12), 1182–1191 (1977).
6. J.M Aloso, L. Magdalena, S. Guillaume, "A hierarchical fuzzy system for assessing interpretability of linguistic knowledge bases in classification problems", Conference Paper, January 2006.

A generator of inclusion measures and embeddings for IVFSs

Michaela Brutenicova[1], Agustina Bouchet[2], Susana Díaz-Vázquez[2], and Susana Montes[2]

[1] Department of Mathematics, Matej Bel University, Slovakia
michaela.brutenicova@umb.sk
[2] Department of Statistics and Operational Research, University of Oviedo, Spain
{bouchetagustina,diazsusana,montes}@uniovi.es

Keywords: interval-valued fuzzy sets· embedding measure· inclusion measure

The development of theories and methodologies for working with imprecision and uncertainty has been an active research area since fuzzy sets were first introduced in 1965 [3]. One particular challenge that researchers have focused on is measuring the degree of inclusion between fuzzy sets, which has led to the study of the axioms needed for defining an inclusion measure [2]. As a result, there are now several definitions of inclusion measures in the literature, each with its own set of axioms.

Fuzzy sets can be seen as a special case of interval-valued fuzzy sets (IVFSs). Some authors have recently proposed new ways to measure IVFSs taking into account the impact of intervals and two important concepts are emerging: "inclusion measures" and "embeddings" [3,1,5]. While these concepts measure different relations between IVFSs, there is a similarity in their conception. However, there are many inclusions, as they are based on the order between the intervals.

In this study, we propose a new measure called the order degree measure on the set of closed subintervals of [0,1], denoted as L([0,1]), that can be used to construct inclusions or embeddings based on the chosen order. We also establish the axioms that the generator must satisfy to construct these measures. By using these order degree measures for intervals, we can obtain decomposable inclusion measures and embedding measures for IVFSs, simply by varying the order on L([0,1]). Thus, we can conclude that inclusions and embeddings are related concepts, but they have different meanings.

Acknowledgements

M. Brutenicova was supported by Erasmus+ KA131 Program.

References

1. Bouchet, A., Sesma-Sara, M., Ochoa, G., Bustince, H., Montes, S., Díaz, I.: Measures of embedding for interval-valued fuzzy sets. Fuzzy Sets and Systems, In press (2023).
2. Bustince, H., Mohedano, V., Barrenechea, E., Pagola, M.: Definition and construction of fuzzy DI-subsethood measures. Information Sciences **176**(21), 3190–3231 (2006).
3. Pekala, B., Dyczkowski, K., Grzegorzewski, P., Bentkowska, U.: Inclusion and similarity measures for interval-valued fuzzy sets based on aggregation and uncertainty assessment. Information Sciences **547**,1182–1200 (2021).
4. Zadeh, L.A.: Fuzzy sets. Information and Control **8**(3), 338–353 (1965).
5. Zeng, W., Guo, P.: Normalized distance, similarity measure, inclusion measure and entropy of interval-valued fuzzy sets and their relationship. Information Sciences **178**(5), 1334–1342 (2008).

An Approach to Refine Time Series Forecast Aggregations Using Ranking Methods and k-Nearest Neighbours

Pelayo Suárez Dosantos, Agustina Bouchet, Irene Mariñas-Collado, and Susana Montes

Department of Statistics and Operation Research and Mathematics Didactics, University of Oviedo, 33007 Oviedo, Spain
{suarezdpelayo,bouchetagustina,marinasirene,montes}@uniovi.es

Keywords: Time series · forecasts · aggregation functions · ranking methods · kNN technique

Time series forecasting is an important task, relevant in many fields, such as finance, economics, and engineering. It involves the use of historical data to predict future values of a variable, such as stock prices, weather patterns, or energy consumption. There are a wide variety of methods available for time series forecasting, including traditional statistical models and more recent machine learning techniques. A sensible choice of the appropriate method is crucial for making accurate predictions. However, every model or technique has its own drawbacks and benefits, and the decision to use a single one rather than several may lead to a poorer prediction. Combining multiple predictions can be an effective way to improve the accuracy and robustness of time series forecasting models by leveraging the strengths of different methods and reducing the impact of potential biases or errors.

Four different strategies are used to obtain predictions for several monthly time series from the M3 Competition [3]. These are: 1) SARIMA (Seasonal AutoRegressive Integrated Moving Average), 2) Holt-Winters, 3) TBATS (Trigonometric seasonality Box-Cox transformation ARMA errors Trend and Seasonal components) and 4) a variation of SVM (Support Vector Machine) regression. The forecasts are then aggregated through the proposed approach.

The suggested method involves the incorporation of three main techniques from diverse areas. This includes: 1) Weighted sums to aggregate the predictions given by different models; 2) Ranking methods to determine appropriate weights for every prediction; 3) Use k Nearest Neighbours (kNN) machine learning technique to determine the best subset of the historical data to make the forecast. More precisely, kNN is applied to search for patterns similar to the observations that precede the times at which predictions shall be made [4]. Once those patterns (time points) are identified, the associated forecasts are considered. The Borda count ranking method is used to rank the four strategies according to how accurate the predictions were at those time points. Based on the count, each technique is given a weight, and the predicted values are then aggregated.

The proposed combination technique is compared with the individual prediction methods forecast accuracy, as well as with other widely used combination procedures such as the arithmetic mean, Bates and Granger [1] and CLS (Constrained Least Squares) [2].

References

1. Bates, J.M., Granger, C.W.: The combination of forecasts. Journal of the operational research society **20**(4), 451–468 (1969)
2. Granger, C.W., Ramanathan, R.: Improved methods of combining forecasts. Journal of forecasting **3**(2), 197–204 (1984)
3. Makridakis, S., Hibon, M.: The m3-competition: results, conclusions and implications. International journal of forecasting **16**(4), 451–476 (2000)
4. Martínez, F., Frías, M.P., Pérez, M.D., Rivera, A.J.: A methodology for applying k-nearest neighbor to time series forecasting. Artificial Intelligence Review **52**(3) (2019)

General convolution operations

Y. Cheng[1,2], B. Zhao[1], L. Zedam[2,3], and B. De Baets[2]

[1] School of Mathematics and Statistics, Shaanxi Normal University, Xi'an, 710119, P.R. China
[2] KERMIT, Department of Data Analysis and Mathematical Modelling, Ghent University, Coupure links 653, B-9000 Gent, Belgium
[3] LMPA, Department of Mathematics, University of M'sila, 28000 M'sila, Algeria E-mails: chengyafei@snnu.edu.cn, zhaobin@snnu.edu.cn, lemnaouar.zedam@ugent.be, bernard.debaets@ugent.be

Convolution operations play an important role in different subfields of engineering and science. In the standard real setting, the convolution of two real functions is computed by means of an integral that expresses the amount of overlap of one function as it is shifted over another function. It holds some interesting mathematical properties such as commutativity, associativity, and distributivity w.r.t. pointwise addition of functions [1].

In black-and-white image processing, convolution-like operations in mathematical morphology are based on Minkowski addition and subtraction of sets. These operations are further generalized in fuzzy mathematical morphology through the use of membership functions and the use of supremum (instead of integration) [2].

Still other similar operations can be encountered in fuzzy logic, where Zadeh's extension principle leads to convolution operations generalizing AND and OR of fuzzy truth values [3,4].

Here, we take a step further by studying two convolution operations on the set of functions between an arbitrary set and a complete lattice. The main question we address is under which conditions the resulting structure is a lattice, extending the results in [5].

To that end, we first study the mathematical properties of these convolution operations, such as commutativity, associativity, idempotence, and so on. We will show that for each of these convolution operations the set of functions is a lattice-ordered monoid w.r.t. pointwise join and meet under some restrictions. Finally, we will study the lattice structure of the set when equipped with the convolution operations.

References

1. S. W. Smith, The Scientist and Engineer's Guide to Digital Signal Processing, California Technical Publishing, 1997, pp. 123-140, Ch. 7-Properties of convolution.
2. J. Serra, Image Analysis and Mathematical Morphology, Academic Press, Inc., Orlando, FL, USA, 1983.
3. D. Dubois, H. Prade, Possibility Theory. An Approach to Computerized Processing of Uncertainty, Springer US, 1988.
4. G. De Cooman, Fuzzy Set Theory and Advanced Mathematical Applications, Springer US, Boston, MA, 1995, pp. 89-133, Ch. 4-Towards a possibilistic logic.
5. L. De Miguel, H. Bustince, B. De Baets, Convolution lattices, Fuzzy Sets and Systems, 335 (2018) 67-93.

Fuzzy Modeling in Solving Volterra Integral Equation with Weakly Singular Kernel

Irina Perfilieva[1] and Thi Minh Tam Pham[2]

[1] Institute for Research and Applications of Fuzzy Modeling, University of Ostrava, 30. dubna 22, 701 03 Ostrava, Czech Republic
[2] Department of Mathematics, Faculty of Science, University of Ostrava, 30. dubna 22, 701 03 Ostrava, Czech Republic
{Irina.Perfilieva,thiminhtam.pham}@osu.cz

Fuzzy modeling has long gone beyond its exclusive use in problems with uncertainty and has become an important tool for the numerical solution of classical problems. The specificity of the proposed methods lies in their robustness to various changes in the input parameters. This property is especially important when solving differential and integral equations with initial data of a more general form than is required when using traditional numerical methods.

Volterra integral equations belong to the group of classical fundamental equations that generate many special cases being of considerable interest for applications: population growth dynamics, epidemics, the study of viscoelastic materials, heat flows, etc.

The general form of the *linear Volterra equation of the second kind* has the form

$$u(t) = g(t) + \lambda \int_a^t K(t,s)u(s)ds, \qquad t \in [a,b], \tag{1}$$

where $\lambda \neq \infty$, and the functions: *kernel* K : [a, b] x[a, b] →R, *source* g: [a, b] →R are given, and u: [a, b] →R is unknown. The main result of the theory of Volterra equations of the second kind states: for each $\lambda \neq \infty$ there exists a square-integrable solution of the Volterra equation of the second kind, and it is unique.

In the modern theory of Volterra equations, kernels other than square-integrable ones are considered. We will focus on *weak singular* kernels, which are mainly described by the form $K(t,s) = \alpha(t,s)|t-s|^{-\nu}$, where $0 < \nu < 1$. Generalizing, we say that a kernel K is weakly singular [1], if it is absolutely integrable with respect to s and satisfies $\sup_{[a,b]} \int_a^b |K(t,s)\, ds| < \infty$.

In most cases, it is not possible to find an exact solution to the Volterra integral equation, so many numerical methods have been developed to find an approximate solution. In this regard, we mention methods that use (i) numerical integration, on the one hand, and (ii) approximation of integrands, on the other. Note that their joint use in combined methods is possible.

In the case of an equation of type (1) with a weakly singular kernel the main problem in the development of numerical methods is the *discontinuity* of the kernel on the diagonal of the integration domain.

In this contribution, we show that in this particular case the (fuzzy) *F-transform* method [2] is applicable and occupies a special place among other combined methods.

In more detail, to find a numerical solution of (1), all involved functional objects are replaced by their approximations in the form of inverse F-transforms. After this step, integration in (1) becomes applicable only to the basic functions of a uniform fuzzy partition, which does not depend on any particular type of integrand. As a result, the action of integration in (1) is focused on what can be associated with a certain spatial structure (fuzzy partition), and separated from the functional objects in (1). The F-transform image of this action is represented as a product of the operational matrix of the Volterra operator and the vector of basic functions of the fuzzy partition. This separation is the main feature that contributes to the efficiency of the proposed method. An additional argument in favor of the F-transform method, which contributes to its low computational complexity, is the triangular form of the operating matrix of the Volterra operator. This leads to a triangular matrix of the resulting system of linear equations and hence to a direct iterative process of finding a solution called *direct substitution*.

Below we demonstrate the above theoretical reasoning on two examples of equation (1), where weakly singular kernels are represented as follows:

$$K_1(t,s) = (t-s)^{-1/3}, \quad K_2(t,s) = (t-1)(t-s)^{-1/3}.$$

Other parameters are: a=0, b=1, $\lambda_1 = \lambda_2 = 1/10$; function g_1 is chosen such that (1) has the exact solution $u_1(t) = (t(1-t))^2$, while $g_2(t) = 1 + t^4$.

The first example was considered in [3], where the proposed numerical method uses the expansion of an unknown function in a Taylor series. The authors were only able to use a Taylor polynomial of degree zero

because approximation with higher degree polynomials requires hypersingular kernel derivatives. The proposed new method, based on inverse F-transform approximations, easily computes an approximate solution with the accuracy errors between 0.0006 and 0.0115.

The exact solution of the second example cannot be expressed analytically. We have chosen this example from [4], where the proposed numerical method uses the 2nd-order N-point spline collocation with grid node optimization. The proposed new method based on the inverse F-transform approximation was applied without optimizing the choice of partition nodes. The obtained results up to the constant multiplier are fully comparable with the much more sophisticated method in [4].

In conclusion, we have proposed a new numerical method based on fuzzy modeling and F-transforms for finding approximate solutions to linear Volterra integral equations with weakly singular kernels. Its usefulness and effectiveness are shown theoretically and with examples.

References

1. Vainikko, G.: Multidimensional weakly singular integral equations. Springer, Berlin (1993).
2. Perfilieva, I.: Fuzzy transform: theory and applications, Fuzzy Sets and Systems 157, 993-1023 (2006).
3. Maleknejad, K., Aghazadeh, N.: Numerical solution of Volterra integral equations of the second kind with convolution kernel by using Taylor-series expansion method, Applied Mathematics and Computation 161, 915–922 (2005).
4. Alijani, Z., Kangro, U.: Numerical solution of a linear fuzzy Volterra integral equation of the second kind with weakly singular kernels, Soft Computing, 26, 12009–12022 (2022).

Performance of Methods for Detection of Structural Breaks in Time Series

Phuong Truong and Vilem Novak

Institute for Research and Applications of Fuzzy Modeling 30. dubna 22 701 03 Ostrava 1, Czech Republic
E-mail: {phuong.truong,vilem.novak}@osu.cz

Keywords: Time series · Fuzzy transform · Evaluative linguistic expressions · Fuzzy natural logic · Structural Break Detetion · Statistic methods.

The primary objective of this research is to evaluate and compare effectiveness of several methods for finding structural breaks in time series. We will consider statistical as well as non-statistical ones. Chow test (1960) in [4], [5] is a statistical method for identification of structural breaks in time series. It is based on the idea that the structural break separates the data into two subsets, each of which can be approximately modeled using a linear function. Hence, different regression coefficients and intercept terms are taken into account. The method considers a null hypothesis of “no structural break" and compares it with the alternative hypothesis “a structural break exists". Pettitt's method developed in 1979 is statistical and is typically utilized when trying to identify a single change point in a time series. It tests the null hypothesis: “the variables follow one or more distributions that have the same location parameter (no changepoint)". The Buishand statistical test (1982) is based on the assumption that the data follow a normal distribution. The data are distributed arbitrarily and independently in accordance with the null hypothesis “no break". This test is particularly sensitive to disruptions in the center of the time series. A non-statistical method for discovering structural breaks in time series is proposed in [2]. It is based on the combination of special techniques of fuzzy modeling, namely the Fuzzy Transform (F-transform) and selected methods of Fuzzy Natural Logic (FNL). In [7] it is suggested to combine it with the Chow test. Presence of the structural break is tested by means of a null hypothesis:

$$H_0 : \beta_k^0[X] = \beta_{k+1}^0[X], \beta_k^1[X] = \beta_{k+1}^1[X]$$

where $k, k+1$ are subscripts of the basic functions A_k, A_{k+1} from the corresponding fuzzy partition. The fuzzy transform applied to time series X generates zero degree (absolute) components $F_k^0[X]$, and first degree ones $F_k^1[X] = \beta_k^0[X] + \beta_k^1[X] \cdot (x - c_k)$. The coefficients $\beta_k^1[X]$ are estimations of average values of the tangents (slopes) of X over areas characterized by the fuzzy sets A_k, $k = 1, \ldots, n$. A structural break is identified in the area covered by $A_k, A_{k'}$ if the value $\beta_k^1[X]$ is “big" and $\beta_{k+1}^1[X]$ is “small” (or vice versa) where the latter evaluative expressions are assigned using the methods of FNL. In this paper we will compare our method with the other statistical ones. In order to evaluate and compare the efficiency and accuracy of all methods, we will apply them to one specific real time series, that includes both break points as well as intervals. The results will be extensively discussed.

References

1. V. Novak, I. Perfilieva, and A. Dvorak: *Insight into Fuzzy Modeling.* Hoboken, NJ, John Wiley & Sons, 2016.
2. Vilem Novak: *Detection of Structural Breaks in Time Series.* International Journal of Fuzzy Logic and Intelligent Systems, (18)2018, 1–12.
3. Vilem Novak, Soheyla Mirshahi, Viktor Pavliska: *LFL Forecaster: Analysis, Forecasting and Mining information from time series.* 2019 IEEE International Conference on Fuzzy Systems, FUZZ-IEEE 2019, New Orleans, LA, USA, June 23-26, 2019, 1–6
4. Bent Nielsen, Andrew Whitby: *A Joint Chow Test for Structural Instability.* Econometrics 3, 2012.
5. Shalabh, IIT Kanpur: *Chapter 12: Tests for Structural Change and Stability.* Econometrics.
6. Ana F.Militino, Mehdi Moradi, M. Dolores Ugarte : *On the Performances of Trend and Change-Point Detection Methods for Remote Sensing Data.* Remote Sens 2020, 12(6), 1008.
7. Vilem Novak, Phuong Truong: *A combination of fuzzy techniques and Chow test to detect structural breaks in time series.* MDPI Axioms (12)2023, 1-17.

Exploring the Impact of Voter Preferences on the Kemeny Distance

Noelia Rico[1], Agustina Bouchet[2], and Irene Díaz[1]

[1] Department of Computer Science, University of Oviedo, Spain
{noeliarico,sirene}@uniovi.es
[2] Department of Statistics and Operation Research and Mathematics Didactics,
University of Oviedo, Spain. bouchetagustina@uniovi.es

Keywords: Ranking aggregation · Distribution of preferences · Computational social choice

The Kemeny method for ranking aggregation is a well-known technique for generating a collective ranking from individual preferences. It is based on determining the winner as the closest to all the opinions, measuring this with a distance based on the number of disagreements between the rankings. However, it is computationally complex, making it difficult to scale to large datasets.

The Kemeny distance is computed using the pairwise comparison of the alternatives in terms of number of votes. This is gathered in a matrix, known as the pairwise comparison matrix. However, when this matrix is obtained from rankings, the distribution of the matrix is restricted. In this work, we study how this impacts on the possible distances obtained for the Kemeny ranking. By analyzing the characteristics of the ranking profile, we are able to identify how these restrictions affect the final distance obtained and the implications for reducing the execution time required to determine the winner. Additionally, we explore the use of the Borda ranking as an initial bound for the Kemeny ranking, and introduce two new initialization bounds for the algorithm. The research also shows that the theoretical bounds are affected by the characteristics of the ranking profile. Furthermore, the study investigates the relationship between the number of voters and the distance between the Borda and Kemeny rankings, and introduces two new initialization bounds for the algorithm. The research also shows that the theoretical bounds are affected by the characteristics of the ranking profile. Additionally, other initialization methods are studied and indices that measure different characteristics of the profile are considered and studied in relation to the final distance obtained. By studying the distance of the winner solution attending to characteristic of the preferences expressed by the voters, this work aims to make the Kemeny method more accessible for real-world applications by reducing the execution time required to determine the winner.

References

1. Brandt, F., Conitzer, V., Endriss, U., Lang, J., & Procaccia, A. D. (2016). Handbook of Computational Social Choice.
2. Rico, N., Vela, C. R., & Díaz, I. (2023). Reducing the time required to find the Kemeny ranking by exploiting a necessary condition for being a winner. European Journal of Operational Research, 305(3), 1323–1336. doi:10.1016/j.ejor.2022.07.031

A metric to evaluate linguistic consensus-reaching processes

D. García-Zamora, Á. Labella, R. M. Rodríguez, and L. Martínez

Universidad de Jaén

Nowadays, real-world decision-making problems become more and more complex and demand Group Decision-Making (GDM) schemes that involve various participants with different perspectives to select the best solution for the problem. In such schemes, conflicts and opinion polarization are usual. Therefore, Consensus Reaching Processes (CRPs) have emerged to deal with such conflicts among the group members and reach an agreed-upon solution. Linguistic Group Decision-Making (LiGDM) and Linguistic Consensus Reaching Processes (LiCRPs) have become popular, and many different proposals for LiGDMs and LiCRPs with fuzzy preference modeling have been introduced in the specialized literature, because of their ability to allow participants to express their opinions by using qualitative terms from natural/artificial languages and managing uncertainty. However, there is a lack of objective metrics to compare and analyze the performance of different LiCRPs in each specific LiGDM problem [1]. Therefore, this contribution introduces the first linguistic metric that objectively compares LiCRPs models and determines the best-performing one [2]. This metric compares the results of a LiCRP with an ideal scenario in which the decision-makers reach the consensus threshold by changing as less as possible their original opinions. To model such an ideal situation, minimum cost consensus models are extended to manage ELICIT (Extended Comparative Linguistic Expressions with Symbolic Translation) information. ELICIT values enable modeling fuzzy linguistic preferences and performing precise computing with words operations.

Keywords: Computing with words, ELICIT information, Fuzzy linguistic approach, Linguistic cost metric, Minimum cost consensus

Acknowledgments

This work is partially supported by ProyExcel_00257, linked to the Andalucía Excellence Research Program, and the Postdoctoral fellow Ramón y Cajal (RYC-2017-21978), the FEDER-UJA project 1380637 and ERDF, by the Spanish Ministry of Science, Innovation and Universities through a Formación de Profesorado Universitario grant (FPU2019/01203) and by the Junta de Andalucía, Andalusian Plan for Research, Development, and Innovation (POSTDOC 21-00461).

References

[1] D. García-Zamora, Á. Labella, W. Ding, R. M. Rodríguez and L. Martínez, "Large-Scale Group Decision Making: A Systematic Review and a Critical Analysis," in IEEE/CAA Journal of Automatica Sinica, vol. 9, no. 6, pp. 949-966, June 2022, doi: 10.1109/JAS.2022.105617.

[2] D. García-Zamora, Á. Labella, R. M. Rodríguez and L. Martínez, "A Linguistic Metric for Consensus Reaching Processes based on ELICIT Comprehensive Minimum Cost Consensus Models," in IEEE Transactions on Fuzzy Systems, 2022, doi: 10.1109/TFUZZ.2022.3213943.

Selection of Circular Economy Indicators through a Large-scale Comprehensive Minimum Cost Consensus Model

Á. Labella, D. García-Zamora, R. M. Rodríguez, and L. Martínez

Department of Computer Science, University of Jaén, Jaén, Spain

Since the publication of the first report on the Circular Economy (CE) in 2013, there has been a surge of interest in the topic from both society and the business community. This has resulted in the development of a substantial body of academic literature aimed at establishing principles that can serve as a theoretical foundation for the CE concept. Governments are seeking to understand how organizations are transitioning to the new production model. However, despite the efforts of researchers and companies to create effective measurement systems, it remains challenging to determine which aspects to measure and how intensely an organization is implementing the CE model. The existing measurement proposals rely on costly and time-consuming methodologies that combine different approaches [1]. To address this issue, we propose a comprehensive consensus model for large-scale group decision-making, which minimizes costs and adjusts experts' initial preferences to obtain accurate measurements of indicators on which all parties can agree. According to the agreement achieved and different rules, the indicators can be accepted or rejected. In this sense, the use of fuzzy thresholds in the acceptance/rejection rules can provide a more flexible selection process. Our research aims not only to provide a fast, useful, and accurate method for measuring CE but also to demonstrate its benefits and effectiveness by comparing its performance to a real-world case in the building industry.

Acknowledgments
This work is partially supported by ProyExcel_00257, linked to the Andalucía Excellence Research Program, and the Postdoctoral fellow Ramón y Cajal (RYC-2017-21978), the FEDER-UJA project 1380637 and ERDF, by the Spanish Ministry of Science, Innovation and Universities through a Formación de Profesorado Universitario grant (FPU2019/01203) and by the Junta de Andalucía, Andalusian Plan for Research, Development, and Innovation (POSTDOC 21-00461).

References
[1] R. M. Rodríguez, Á. Labella, P. Nunez-Cacho, V. Molina-Moreno, and L. Martínez. "A comprehensive minimum cost consensus model for large scale group decision making for circular economy measurement". Technological Forecasting and Social Change, 2022, 175, 121391.

Similarities between General Type-2 Fuzzy Sets

Pedro Huidobro[1], Hani Hagras[2], Javier Andreu-Perez[2], Humberto Bustince[3], and Pedro Alonso[1]

[1] University of Oviedo, Oviedo, Spain
huidobropedro@uniovi.es, palonso@uniovi.es
[2] Centre for Computational Intelligence, School of Computer Science and Electronic Engineering (CSEE), University of Essex, Colchester, UK
hani@essex.ac.uk, javier.andreu@essex.ac.uk
[3] Institute of Smart Cities (ISC), Department of Statistics, Computer Science and Mathematics, Public University of Navarre (UPNA), Pamplona, Spain
bustince@unavarra.es

Keywords: Interval type-2 fuzzy sets · Interval-valued fuzzy sets · Similarity measures

Fuzzy sets were first established by Zadeh[3] in 1965. In 1975, he also proposed the concept of type-n fuzzy sets[4], where the membership function is a type(n-1) fuzzy set. Interval type-2 fuzzy sets (IT2FSs)[2] are a particular case of type-2 fuzzy sets where the secondary membership function is equal to 1 in a subset of the domain and 0 otherwise. There are several examples of interval type-2 fuzzy sets that the majority of the authors did not take into account[1].

Nowadays, it is quite common to employ similarity measures where the most common measures employ a real-valued function, where 0 indicates complete dissimilarity and 1 indicates identical.

In our paper, we examined the literature for similarity measures for IT2FSs and discovered that most of them have certain shortcomings, mainly that they do not work properly for non convex secondary membership functions. In this work, we will present some counter-examples and propose a new similarity based on Jaccard's similarity which allows handling non convex secondary membership functions.

Acknowledgements

P. Huidobro is supported by the grant program (PA-20-PF-BP19-169), H. Bustince by the project PID2019-108392 GB I00 (AEI/10.13039/ 501100011033) and P. Alonso by the program of the Spanish Ministry of Science and Technology (TIN-2017-87600-P).

References

1. H. Bustince Sola, J. Fernandez, H. Hagras, F. Herrera, M. Pagola, and E. Barrenechea. Interval type-2 fuzzy sets are generalization of interval-valued fuzzy sets: Toward a wider view on their relationship. *IEEE Transactions on Fuzzy Systems*, 23(5):1876–1882, 2014.
2. J. McCulloch, C. Wagner, and U. Aickelin. Extending similarity measures of interval type-2 fuzzy sets to general type-2 fuzzy sets. In *2013 IEEE international conference on fuzzy systems (FUZZ-IEEE)*, pages 1–8. IEEE, 2013.
3. L. A. Zadeh. Fuzzy sets. *Information and Control*, 8(3):338 – 353, 1965.
4. L. A. Zadeh. The concept of a linguistic variable and its application to approximate reasoning—I. *Information Sciences*, 8(3):199 – 249, 1975.

On disimilarities between IVFSs defined from disimilarities between fuzzy sets

Emilio Torres-Manzanera, Agustina Bouchet, Susana Díaz-Vázquez, and Susana Montes

Department of Statistics and Operational Research,
University of Oviedo, Spain {torres,bouchetagustina,diazsusana,montes}@uniovi.es

Keywords: interval-valued fuzzy sets · fuzzy sets · dissimilarity

The search for a measure that defines (dis)similarity between interval-valued fuzzy sets (IVFSs) has received great attention in the last two decades. The first proposals that we can find in the literature were functions that provided as a final result a unique value (see for example [2]). This makes sense when the information is provided by fuzzy sets. If the degree of membership of the elements can be expressed in such a precise way that it is just one value, the difference between two opinions (difference between two fuzzy sets) can be expected to be provided with precision (should also be given by just one value). However, when the experts cannot be so precise and are allowed to provide an interval to express the membership of each element to a set, the differences between experts should also be allowed to be imprecise: an interval is the reasonable output in general [1,3,4,5]. But we should not forget that as particular cases of IVFSs we have fuzzy sets. In other words, that even in the context of IVFSs we allow judges to be accurate if they can and that in this case the degree of disagreement could/should be required to be precise as well.

In this contribution we rethink the definition of (dis)similarity between IVFSs focusing on the fact that fuzzy sets are particular cases of IVFSs. We first establish an axiom for the dissimilarity between "degenerated IVFSs", this is, between fuzzy sets. Departing from this axiom, we revisit different properties that are usually included in the definition of dissimilarity between IVFSs and obtain new conditions that follow from combining our departing premise with other axioms of well-known previous definitions. Most of these new properties being expressed in terms of the comparison of the fuzzy sets that are included in an interval-valued fuzzy set.

References

1. Bustince, H., Marco-Detchart, C., Fernández, J., Wagner, C., Garibaldi, J.M., Takáč, Z.: Similarity between interval-valued fuzzy sets taking into account the width of the intervals and admissible orders. Fuzzy Sets and Systems **390**:23–47, 2020.
2. Grzegorzewski, P.: Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the hausdorff metric. Fuzzy Sets and Systems **148**(2):319–328, 2004.
3. Pekala, B., Dyczkowski, K., Grzegorzewski, P., Bentkowska, U.: Inclusion and similarity measures for interval-valued fuzzy sets based on aggregation and uncertainty assessment. Information Sciences **547**:1182–1200, 2021.
4. Takáč, Z., Bustince, H., Pintor, J. M., Marco-Detchart, C., Couso, I.: Width-based interval-valued distances and fuzzy entropies. IEEE Access **7**:14044–14057, 2019.
5. Torres-Manzanera, E., Král, P., Janiš, V., Montes, S.: Uncertainty-aware dissimilarity measures for interval-valued fuzzy sets. International Journal of Uncertainty, Fuzziness and Knowlege-Based Systems **28**(5):757 – 768, 2020.

Interactive computing

Jiří Kupka

CE IT4I IRAFM, University of Ostrava 30. dubna 22, 701 03 Ostrava, Czech Republic
Jiri.Kupka@osu.cz
http://ifm.osu.cz/

Keywords: interactivity · fuzzy arithmetics · joint possibility distribution.

The topic of this contribution belongs to the basic ground of fuzzy set theory. We plan to discuss the notion of interactivity. Although the notion of interactivity is quite old (It has been defined by L. Zadeh in 1975.), many mathematicians do not consider it when applying some results of fuzzy set theory. This fact is somewhat surprising because the well-known Zadeh's (extension) extension is a special case of the interactivity-based (sup-J) extension principle.

The notion of interactivity between two or more fuzzy numbers is strongly connected with notions of a joint possibility extension J and a so-called sup-J extension principle, and it has been intensively studied in the last two decades. For instance, in 2004, Fullér, Carlsson, and Majlender ([1]) introduced a relation between interactivity and a joint possibility extension, practical aspects of interactive computing (the one given by a sup-J extension principle) were studied by K. Scheerlinck, B. de Baets, et al. ([2]) about ten years ago, and numerous mathematicians studied also interactive fuzzy arithmetic in the last decade. Surprisingly, the interactive arithmetics can provide some interesting features, which are not available for "standard" fuzzy arithmetic (i.e. the one using Zadeh's extension principle), and the group around E. Esmi, de Barros et al. recently showed useful practical impacts of the interactive computing. For instance, one can mention the existence of the inverse element for the interactive addition, the existence of the interactive derivative, etc.

In our talk, we would like to go further in this direction, by studying some aspects of interactive arithmetics: for instance, by studying conditions, under which the interactive operation preserves inverse elements of given operations.

References

1. C. Carlsson, R. Fuller, P. Majlender, *Additions of completely correlated fuzzy numbers*, IEEE International Conference on Fuzzy Systems, vol. 1 (2004), 535–539.
2. K. Scheerling, H., Vernieuwe, B. De Baets, *Zadeh's extension principle for continuous functions of non-interactive variables: a parallel optimization approach*, IEEE Trans. Fuzzy Syst. 20 (1) (2012), 96–108.

Fuzzy rules weightening given by implicative GUHA quantifiers

Martina Daňková

University of Ostrava, Dvořákova 7, 701 03 Ostrava
martina.dankova@osu.cz
https://ifm.osu.eu/

Weighted fuzzy rules are commonly utilized in fuzzy logic systems as they enhance the system's output accuracy and reliability. Fine-tuning of the behavior and sensitivity of the system to different input conditions is feasible by adjusting the weighting factors of the fuzzy rules at various levels, including the antecedent level, the consequent level or the whole rule (see, e.g., [1]).

In this contribution, we employ a novel approach using generalized quantifiers that are appropriate for analyzing dependencies in the form of "If antecedent, then consequent," which are known as implicational quantifiers of the General Unary Hypothesis Automaton (GUHA) method [2]. Subsequently, we combine the values of this quantifier, calculated on the basis of input observations, with suitable fuzzy rules analogously to the case of implicative normal forms introduced in [3]. This approach leads to a novel category of weighted fuzzy rules. Finally, we provide some examples that illustrate its behavior and suitability for data analysis.

Acknowledgment

This research was supported by the Czech Science Foundation project No. 23-06280S.

References

1. de la Ossa, L., Gamez, J.A., Puerta, J.M.: Learning weighted linguistic fuzzy rules by using specifically-tailored hybrid estimation of distribution algorithms. International Journal of Approximate Reasoning 50(3), 541–560 (2009).
2. Hájek, P., Holeňa, M., Rauch, J.: The GUHA method and its meaning for data mining. Journal of Computer and System Sciences 76(1), 34–48 (2010).
3. Daňková, M.: Approximation of extensional fuzzy relations over residuated lattices. Fuzzy Sets and Systems 161(14), 1973 – 1991 (2010).

On generalized quantifiers in multi-adjoint logic programming

Jesús Medina, José Antonio Torné-Zambrano

Department of Mathematics. University of Cádiz. Spain
{jesus.medina, joseantonio.torne}@uca.es

Logic programming is a relevant field in knowledge representation, which has been widely studied from both theoretical and applied perspectives. The semantics of a logic program has been characterized by the post-fixed points of the immediate consequences operator in a fuzzy framework [2]. This operator is defined from the supremum of a set of values. Hence, the existential feature of the supremum is inherent in its definition. Therefore, when for example the dataset contains some noise, the final computed result (the output of the immediate consequences operator, and so the semantics associated with the given logic program) can be altered with only one wrong value in the computation. In order to solve this problem, in [3] a broader definition of the immediate consequences operator was presented based on generalized quantifiers, which main properties were studied in [4].

This paper advances in this challenge introducing a new definition of orness [5] adapted to this framework. This notion is fundamental to be sure that the introduced immediate consequences operator defined through generalized quantifiers is not far from the original definition based on the supremum operator, that is, the operator which has the greatest orness. Moreover, novel theorems on the continuity of the immediate consequences operator will be introduced. Ensuring the continuity of this operator, as Damásio and Pereira asserted in [1], is paramount in order to obtain the least fixed point (the optimal semantics) of the program in a countable number of iterations.

Acknowledgements Partially supported by the 2014–2020 ERDF Operational Programme in collaboration with the State Research Agency (AEI) in project PID2019-108991GB-I00, with the Ecological and Digital Transition Projects 2021 of the Ministry of Science and Innovation in project TED2021- 129748B-I00, and with the Department of Economy, Knowledge, Business and University of the Regional Government of Andalusia in project FEDER-UCA18-108612, and by the European Cooperation in Science & Technology (COST) Action CA17124.

References

[1] C. V. Damásio and L. M. Pereira. Hybrid probabilistic logic programs as residuated logic programs. In Logics in Artificial Intelligence, JELIA'00, pages 57–73. Lecture Notes in Artificial Intelligence 1919, 2000.
[2] J. Medina, M. Ojeda-Aciego, and P. Vojtáˇs. Multi-adjoint logic programming with continuous semantics. In Logic Programming and Non-Monotonic Reasoning, LPNMR'01, pages 351–364. Lecture Notes in Artificial Intelligence 2173, 2001.
[3] J. Medina and J. A. Torné. Fuzzy Logic Programming with Generalized Quantifiers, pages 17–23. Springer International Publishing, Cham, 2023.
[4] J. Medina and J. A. Torné-Zambrano. Immediate consequences operator on generalized quantifiers. Fuzzy Sets and Systems, 2022.
[5] R. Yager. On ordered weighted averaging aggregation operators in multicriteria decision making. IEEE Transactions on Systems, Man and Cybernetics, 18:183–190, 1988.

Closure Structures as fixed points of some Galois connections

Manuel Ojeda-Hernández, Inma P. Cabrera, Pablo Cordero, and Emilio Muñoz-Velasco

Universidad de Málaga, Andalucía Tech, Málaga, Spain
{manuojeda,ipcabrera,pcordero,ejmunoz}@uma.es

The starting point of this work is [2], where the fuzzy powerset of a fuzzy lattice A, the set of isotone mappings on A and the set of isotone total relations on A were proved to be related by three fuzzy Galois connections such that fuzzy closure systems, fuzzy closure operators and the so called strong fuzzy closure relations are fixed points. The final part of that paper studied the commutativity of the diagrams formed by these mappings.

The next step would be to include closure systems as crisp sets in this problem. Since the powerset of A is a partially ordered set, this addition might be done in two main ways, either we consider the 1-cut of the preposets and study the crisp problem, or consider the "fuzzification" of the crisp order relation.

The restriction of the results in [2] to the 1-cut behave properly within the new paradigm and we focus on the definition of the two fuzzy Galois connections between $(2^A, \subseteq)$ and $(\mathrm{Ext}(L^A), S)$, and $(2^A, \subseteq)$ and $(\mathrm{Isot}(A^A), \preceq)$, where $\preceq$ is the pointwise order. These two conjectures hold substituting the set of all fuzzy sets by the set of all extensional fuzzy sets, which is not a strong restriction since every fuzzy closure system is an extensional set. This study was carried out in [1], which is currently submitted to journal. The study of the commutativity of the whole diagram of fuzzy Galois connections is still an open problem. Some partial solutions to this problem are the restriction to the fuzzy closure structures and the use of a Heyting algebra as the underlying residuated lattice.

References

1. M. Ojeda-Hernández, I. P. Cabrera, P. Cordero, and E. Muñoz-Velasco. Fuzzy closure structures as formal concepts II. *Submitted to Fuzzy Sets and Systems*.
2. M. Ojeda-Hernández, I. P. Cabrera, P. Cordero, and E. Muñoz-Velasco. Fuzzy closure structures as formal concepts. *Fuzzy Sets and Systems*, 2022. doi:10.1016/j.fss.2022.12.014.

This research is partially supported by the FPU19/01467 (MCIU) internship and the research project PID2021-127870OB-I00 (MCIU/AEI/FEDER, UE)

On the problem for ordering Z-numbers based on discrete fuzzy numbers

Arnau Mir-Fuentes[1], Laura De Miguel[1], Sebastia Massanet[2,3], Arnau Mir[2,3], and Juan Vicente Riera[2,3]

[1] Dept. of Estadística, Informática y Matemáticas, Universidad Pública de Navarra
{arnau.mir,laura.demiguel}@unavarra.es
[2] Soft Computing, Image Processing and Aggregation (SCOPIA) Research Group, Dept. Mathematics and Computer Science, University of the Balearic Islands, Palma, Spain
{s.massanet,arnau.mir,jvicente.riera}@uib.es
[3] Health Research Institute of the Balearic Islands (IdISBa), Palma, Spain

In 2011, L. Zadeh introduced the computational linguistic model based on Z-numbers with the intention of modelling natural language more accurately [6]. A Z-number is a pair (A, B) of fuzzy numbers where the second component expresses the credibility/confidence/assurance about the value taken by the first. However, the literature shows that their use is associated with high computational costs, an important drawback for any application. A new approach has recently been proposed in [3,4] that considers Z-numbers based on discrete fuzzy numbers (DFNs) to reduce the computational costs while still maintaining the inherent linguistic flexibility of these operators.

In this work, and following the ideas established in [3], the construction of total orders on the set of Z-numbers based on DFNs is investigated. Specifically, the total order is designed for Z-numbers based on DFNs whose membership values belong to a finite set. The method relies on solid and coherent linguistic criteria and several linguistic properties are analyzed. Indeed, the first components of the Z-numbers to order are transformed by using the credibility of the second components in the sense that a low credibility enlarges the uncertainty of the first component. These transformations generate DFNs to which an admissible order defined on the set of these operators [5] is applied by using different linear interval orders as established in [1,2] to compare their cuts. The paper concludes by presenting an illustrative example to show the applicability of this approach in a decision making problem by means of the ordering of the Z-numbers provided by the experts.

Acknowledgements

This work was supported by Navarra de Servicios y Tecnologías and by the Spanish Ministry of Science (PID2019-108392GB-I00 AEI/10.13039/501100011033). S. Massanet, A. Mir and J.V. Riera acknowledge the partial support of the R+D+i Project PID2020-113870GB-I00-"Desarrollo de herramientas de Soft Computing para la Ayuda al Diagnóstico Clínico y a la Gestión de Emergencias (HESOCODICE)", funded by MCIN/AEI/10.13039/501100011033/.

References

1. Bustince, H., Fernandez, J., Kolesárová, A., Mesiar, R.: Generation of linear orders for intervals by means of aggregation functions. Fuzzy Sets and Systems **220**, 69–77 (2013), Theme: Aggregation functions
2. De Miguel, L., Bustince, H., Fernandez, J., Induráin, E., Kolesárová, A., Mesiar, R.: Construction of admissible linear orders for interval-valued Atanassov intuitionistic fuzzy sets with an application to decision making. Information Fusion **27**, 189–197 (2016)
3. Massanet, S., Riera, J.V., Torrens, J.: A new vision of Zadeh's Z-numbers. In: Carvalho, J.P., Lesot, M.J., Kaymak, U., Vieira, S., Bouchon-Meunier, B., Yager, R.R. (eds.) Information Processing and Management of Uncertainty in Knowledge-Based Systems. pp. 581–592. Springer International Publishing, Cham (2016)
4. Massanet, S., Riera, J.V., Torrens, J.: A new approach to Zadeh's Z-numbers: Mixed-discrete Z-numbers. Information Fusion **53**, 35–42 (2020)
5. Riera, J.V., Massanet, S., Bustince, H., Fernandez, J.: On admissible orders on the set of discrete fuzzy numbers for application in decision making problems. Mathematics **9**(1) (2021)
6. Zadeh, L.A.: A note on Z-numbers. Information Sciences **181**(14), 2923–2932 (2011)

Parametrized Similarity Measure Based on Interpolative Boolean Algebra

Ana Poledica, Pavle Milošević, Bratislav Petrović, and Ilija Antović

University of Belgrade, Faculty of Organizational Sciences, Jove Ilića 154, 11000 Belgrade, Serbia
{ana.poledica, pavle.milosevic, bratislav.petrovic, ilija.antovic}@fon.bg.ac.rs
http://www.fon.bg.ac.rs

This study continues research on multi-valued logic approach to similarity modeling based on interpolative Boolean algebra (IBA) in [1,1]. A novel parametrized [0,1]-valued logic measure, as an extension of IBA equivalence with parameters, is defined for measuring similarity. Besides the theoretical background and properties checking, in this research special attention is devoted to empirical analysis. For validation purposes, classification based on IBA similarity is used. Defined parametrized measures are evaluated and compared in the case of standard classification data sets, and it is shown that they can improve classification results.

Keywords: Similarity modeling · Interpolative Boolean algebra · IBA equivalence · Parametrized similarity.

References

1. Poledica, A., Milošević, P., Dragović, I., Radojević, D., Petrović, B.: A consensus model in group decision making based on interpolative Boolean algebra. In: Pasi, G., Montero, J., Ciucci, D. (eds.) EUSFLAT 2013, Advances in Intelligent Systems Research, vol. 32, pp. 688–694. Atlantis Press (2013).
2. Poledica, A., Milošević, P., Dragović, I., Petrović, B., Radojević, D.: Modeling consensus using logic-based similarity measures. Soft Computing **19**(11), 3209–3219 (2015)

On analysis of stochastic processes by higher degree F-transform*

Holčapek Michal[1], Nguyen Linh[1], and Rico Agnès[2]

[1] Institute for Research and Applications of Fuzzy Modelling, NSC IT4Innovations,
University of Ostrava, 30. dubna 22, 701 03 Ostrava 1, Czech Republic {michal.holcapek,linh.nguyen}@osu.cz

[2] Entrepôt Représentation et Ingénierie des Connaissances
Université de Lyon, 43 bld du 11 novembre 69100 Villeurbanne, France
agnes.rico@univ-lyon1.fr

The fuzzy transform of higher degree (F-transform for short) is a useful tool in data analysis and processing. Among the most important features of the F-transform belong higher frequency filtering and noise reduction, which was successfully used in time series analysis in [3]. The theoretical analysis demonstrated the applicability of the F-transform for trend-cycle estimation in time series modeling, which, among other things, initiated theoretical research on the F-transform of stochastic processes and time series as their realizations. However, this research is based on an "ad hoc" definition of the F-transform for stochastic processes, where computational formulas with the standard Riemann integral are replaced by formulas with the mean square Riemann integral of the stochastic process (see e.g. [4]). Since the components of the F-transform of functions are orthogonal projections onto subspaces of polynomials in the corresponding weighted Hilbert spaces, a natural question arises whether the F-transform of stochastic processes can be introduced in a similar way. The aim of the talk is to show that an affirmative answer to this question can be achieved using so-called (weighted) Bochner spaces of random processes, where an orthogonal projection leads to a subspace of random polynomials. We show that the previously used definition of "ad hoc" is correct, i.e. the computational formulas have the same form, only the integral changes. We also present an interesting result about autocorrelation functions (see [4]), which says that the F-transform of an autocorrelation function (a complex-valued two-dimensional function) coincides with the F-transform of a stochastic process (a complex-valued one-dimensional stochastic process).

References

1. Perfilieva I.: *Fuzzy transforms: theory and applications.* Fuzzy Sets Syst. **157(8)** (2006) 993–1023.
2. Perfilieva I., Daňková M., Bede B. *Towards a higher degree F-transform.* Fuzzy Sets Syst. **180(1)** (2011) 3–19.
3. Novák V., Perfilieva I., Holčapek M., Kreinovich V.: *Filtering out high frequencies using F-transform.* Inf Sci. **274** (2014) 192–209.
4. Holčapek M., Nguyen, L.: *Analysis of autocorrelation function of stochastic processes by F-transform of higher degree.* Soft Comput. **12** (2021) 7707–7730.

* This work has been supported by the Czech Science Foundation through the project No. 23-06280S.

Relational equations in the framework of Omega algebras

Andreja Tepavčević[1,2]

[1] Mathematical Institute SANU, Kneza Mihaila 36, 11000 Belgrade, Serbia
[2] Department of Mathematics and Informatics, Faculty of Sciences, University of Novi Sad, Trg Dositeja Obradovića 4, 21000 Novi Sad, Serbiaandreja@dmi.uns.ac.rs

An overview of recent results on relational equations on Omega algebras, possible applications and some new perspectives will be presented. The essential part of Omega algebras is a fuzzy weak equivalence relation compatible with algebra operations. Several types of algebras are developed and investigated in this context [1,2,3,4]. Identities are valid approximately (up to the equivalence relation). Omega algebras can be represented by a closure system of the lattice of weak congruences. This is a context in which the equations and relational equations are defined. As an algebra with one binary operation, a quasigroup is the most convenient algebraic structure for solving simple equations and systems of equations. Omega quasigroups and methodology of equations and inequalities with one unknown were developed in [3]. Further, the notion of the Omega ring was developed in [4] as a good framework for developing Omega polynomials over the Omega rings and polynomial equations as the next step in our further investigations. Moreover, approximate solutions of two main types of matrix equations were presented in the context where the operation on matrices is not a standard composition but any convenient binary operation [5]. This approach is further generalized to many different types of relational equations, and some possible areas of applications are proposed. Part of the results was obtained in cooperation with Branimir Seselja, Aleksandar Krapez,

Vanja Stepanovic, Jesus Medina, Jorge Jimenez, Maria Luisa Serrano, Branka Budimirovic, and Vjekoslav Budimirovic.

Keywords: Omega Algebras, Relational Equations, Approximate Solutions.

References

1. Budimirović B., Budimirović, V., Šešelja, B., Tepavčević, A.: Fuzzy identities with application to fuzzy semigroups. Information Sciences 266, 148-159 (2014).
2. Budimirović, B., Budimirović, V., Šešelja, B., Tepavčević A,: E-fuzzy groups. Fuzzy Sets and Systems 289, 94–112 (2016).
3. Krapež, A., Šešelja, B., Tepavčević, A.: Solving linear equations by fuzzy quasigroups techniques. Information Sciences 491, 179–189 (2019).
4. Jimenez, J., Serrano M.L., Šešelja, B., Tepavčević, A.: Omega-rings. Fuzzy Sets and Systems 455, 183-197 (2023).
5. Medina, J., Stepanović, V., Tepavčević, A.: Solutions of matrix equations with weak fuzzy equivalence relations. Information Sciences 629, 634-645 (2023).

Eliciting perceptions on the proximities between linguistic terms through sliders

José Luis García-Lapresta, Rodion Lurev, and David Pérez-Román

Universidad de Valladolid

Keywords: ordered qualitative scales, perceptions,surveys

In many decision problems, individuals evaluate a set of alternatives through ordered qualitative scales instead of using numerical values. The reason is that human beings are more comfortable using words than quantitative assessments. Sometimes it is implicitly assumed that ordered qualitative scales are uniform: the psychological proximity between consecutive terms of the ordered qualitative scale is perceived as identical. However, that assumption could be not realistic in some cases. For instance, in the 5-term ordered qualitative scale terrible, poor, average, good, great, used by the Pew Research Center, if 'good' is perceived closer to 'great' than to 'average', or if 'average' is perceived closer to 'good' than to 'poor', etc., then the scale is not uniform.

In order to deal with that problem, García-Lapresta and Pérez-Román (Applied Soft Computing, 2015) introduce ordinal proximity measures for dealing with non-uniform ordered qualitative scales, taking into account ordinal proximities between the linguistic terms of the scales. García-Lapresta, González del Pozo and Pérez-Román (Information Sciences, 2018) provide an algorithm that generates metrizable ordinal proximity measures from the answers to 2-4 questions about the proximities between the linguistic terms of 4-term ordered qualitative scales. Nevertheless, when the scale has more than 4 terms, there is a high increase in the problem's complexity.

In this contribution, we propose a different procedure to the one included in García-Lapresta, González del Pozo and Pérez-Román (Information Sciences, 2018) to associate a metrizable ordinal proximity measure to an ordered qualitative scale. The new proposal is based on a visual procedure that generates a metrizable ordinal proximity measure on an ordered qualitative scale through sliders. The new procedure has been implemented in some online surveys in order to know how some ordered qualitative scales used by public organizations are perceived by the respondents.

An Approach of Solving Volterra Integro-Differential Equations Using Neural Networks

Zahra Alijani

Institute for Research and Applications of Fuzzy Modeling University of Ostrava

Keywords: Neural Networks, Volterra IDE, boundary value problems

Artificial Neural Networks (ANNs) are machine learning models designed to mimic the structure and function of the human brain. They can solve various problems in various fields, including mathematical problems, financial problems, engineering, and medical sciences. The use of ANNs to solve differential equations, such as Volterra integrodifferential equations, is an active area of research. In this research, we propose a methodology for using ANNs to solve these types of equations and compare the results of this method with those of other well-known procedures to demonstrate its effectiveness. We propose a framework for using artificial neural networks (ANNs) to solve Volterra integrodifferential equations (IDEs). The proposed methodology uses feed-forward neural networks (NNs) as surrogate models to approximate the exact solutions of IDEs. The solutions obtained through this method are differentiable and written in closed analytical forms. After the feed-forward, the network will approach the same solutions and enhance the biases. The training of NNs is based on different optimization algorithms, and a trial solution is used as a generalized solution that satisfies the boundary conditions. We also compare various activation functions and choose the hyperbolic tangent function as the basis activation function.

Part III

AGOP General Track

K-increasing functions and their properties

Radko Mesiar, Anna Kolesárová, and Adam Šeliga

Department of Mathematics and Descriptive Geometry, Faculty of Civil Engineering, Slovak University of Technology in Bratislava, Radlinského 11,
810 05 Bratislava, Slovakia
{radko.mesiar,anna.kolesarova,adam.seliga}@stuba.sk

In this contribution, we introduce K-increasing functions which are a generalization of monotone functions. A monotone function $f\colon I^n \to \mathbb{R}$, where I is a real interval, is either increasing in all variables or decreasing in all variables. In real-world applications, it is not unusual to consider functions that are hybrid monotone, i.e., monotone in all variables, but not in the same sense. As a typical example of such functions, we can mention utility functions which are used in multi-criteria decision-making where positive and negative criteria are processed simultaneously. Consider a function $f\colon I^n \to \mathbb{R}$ and a set $K \subseteq \{1, 2, \ldots, n\}$. We say that the function f is a K-increasing function if and only if it is increasing in the variables indexed by elements of K and decreasing in all variables with indices in $K^{\mathsf{c}} = \{1, 2, \ldots, n\} \setminus K$. An important example of $\{2\}$-increasing functions $f\colon [0,1]^2 \to [0,1]$ is the class of fuzzy implications, which, together with hybrid monotone utility functions, were our main motivation for building the framework of K-increasing functions. In this work, basic properties of K-increasing functions are examined and exemplified. Special attention is devoted to K-increasing aggregation functions which are a generalization of aggregation functions (see, e.g., [1]).

Acknowledgements Radko Mesiar and Adam Šeliga acknowledge the support of the grant APVV-18-0052 and VEGA 1/0036/23. Anna Kolesárová also acknowledges the support of the grant VEGA 1/0468/20.

References

1. Grabisch, M., Marichal, J.-L., Mesiar, R., Pap, E.: Aggregation functions. Cambridge University Press, 2009. ISBN 9781139644150.

Concordance measures - some new constructions

Radko Mesiar[1], Anna Kolesárová[1], and Ayyub Sheikhi[2]

[1] Department of Mathematics and Descriptive Geometry, Faculty of Civil Engineering, Slovak University of Technology in Bratislava, Radlinského 11,
810 05 Bratislava, Slovakia
{radko.mesiar,anna.kolesarova}@stuba.sk
[2] Department of Statistics, Faculty of Mathematics and Computer, Shahid Bahonar University of Kerman, Kerman, Iran
sheikhy.a@uk.ac.ir

In this contribution, we focus on new constructions of concordance measures. In the first part, we consider concordance measures induced by a single point (x, y), $0 < y \leq x \leq \frac{1}{2}$. Note that if $x = y = \frac{1}{2}$, the Blomqvist beta is covered. For some details we recommend our recent paper [2]. Let $\mathcal{C}$ be the class of all bivariate copulas. The introduced concordance measure $\kappa_{(x,y)}$ is for each $C \in \mathcal{C}$ given by $\kappa_{(x,y)}(C) = \frac{K_{(x,y)}(C)-2}{4y}$, where

$$K_{(x,y)}(C) = C(x,y) + C(y,x) + C(1-x,y) + C(x,1-y) + C(y,1-x) \\ + C(1-y,x) + C(1-x,1-y) + C(1-y,1-x),$$

and it is convex, i.e., for all copulas $C_1, C_2 \in \mathcal{C}$ and $\lambda \in [0,1]$ we have

$$\kappa_{(x,y)}\left(\lambda C_1 + (1-\lambda)C_2\right) = \lambda\kappa_{(x,y)}(C_1) + (1-\lambda)\kappa_{(x,y)}(C_2).$$

In the second part, we present and discuss transformed concordance measures $f(\kappa)$, $f\colon [-1,1] \to [-1,1]$ being an odd automorphism. For example, if $f(x) = x^3$ then for any concordance measure $\kappa\colon \mathcal{C} \to [-1,1]$, κ^3 is also a concordance measure. In the third part, we consider n concordance measures $\kappa_1, \ldots, \kappa_n$ and an n-ary aggregation function $A\colon [-1,1]^n \to [-1,1]$. If A is continuous and $A(-\mathbf{x}) = -A(\mathbf{x})$ for each $\mathbf{x} \in [-1,1]^n$, then $\kappa = A(\kappa_1, \ldots, \kappa_n)$ is a concordance measure, too. Finally, we introduce probability-based constructions of concordance measures, covering, among others, Spearman's ρ, Gini's γ and also copula-based constructions due to Fuchs and Schmidt [1].

Acknowledgements: R. Mesiar was supported by the grant VEGA 1/0036/23 and A. Kolesárová thanks for the support of the grant VEGA 1/0468/20. The support of the grant APVV-18-0052 is also acknowledged by both authors.

References

1. S. Fuchs, K. D. Schmidt, Bivariate copulas: transformations, asymmetry and measures of concordance. *Kybernetika* **50** (2014) 109–125.
2. R. Mesiar, A. Kolesárová, and A. Sheikhi, Convex concordance measures. *Fuzzy Sets and Systems* **441** (2022) 366–377.

The Choquet integral based on conditional aggregation operators and sublinear means*

Stanislav Basarik, Lenka Halčinová, and Mária Slovinská

Institute of Mathematics, Faculty of Science, Pavol Jozef Šafárik University in Košice, Jesenná 5, 040 01 Košice, Slovakia
stanislav.basarik@student.upjs.sk, lenka.halcinova@upjs.sk, maria.slovinska@student.upjs.sk

In 2015, Do and Thiele introduced the outer measure spaces theory in the context of harmonic and time-frequency analysis in the paper [2]. They constructed a new type of nonadditive integral using the so-called size and the outer essential supremum related to it. This concept became the basis for further research. Several authors developed their ideas. In [3], the authors generalized the original concept by considering arbitrary monotone set functions instead of only subadditive set functions. Later, Boczek et al. in [1] generalized this concept by replacing the outer essential supremum with the so-called conditional aggregation operator. The values of this operator do not depend only on the input vector, but also on the (conditional) set. This concept generalizes the standard version of the level measure. The work of Do and Thiele was also studied in the paper [4], in which a generalization of the integral based on sublinear means is presented.

In the contribution, we shall discuss the Choquet integral based on conditional aggregations and sublinear means. Weighted sum, the standard Choquet integral with respect to submeasure, etc. are examples of conditional aggregation operators and also sublinear means. Therefore it is interesting to compare the constructions of these generalizations of the Choquet integral. Namely, we shall present sufficient conditions under which the concept based on sublinear means coincides with the concept based on the conditional aggregation operators. In the contribution, we shall discuss also the basic properties of these concepts.

References

1. Boczek, M., Halčinová, L., Hutník, O., Kaluszka, M.: Novel survival functions based on conditional aggregation operators. Information Sciences **580**, pp. 705–719 (2021).
2. Do, Y., Thiele, Ch.: L^p theory for outer measures and two themes of Lennart Carleson united. Bulletin of the American Mathematical Society **52**(2), pp. 249–296 (2015).
3. Halčinová, L., Hutník, O., Kiseľák, J., Šupina, J.: Beyond the scope of super level measures. Fuzzy Sets and Systems **364**, pp. 36–63 (2019).
4. Pap, E.: Three types of generalized Choquet integral. Bollettino dell'Unione Matematica Italiana **13**, pp. 545–553 (2020).

* Supported by the grants APVV-21-0468, VEGA 1/0657/22, VVGS-PF-2022-2143.

Local linearity of aggregation and related functions

Andrea Stupňanová

STU in Bratislava, Faculty of Civil Engineering, Radlinského 11, Bratislava, Slovakia

Linearization plays a key role in several models of real life problems. We focus on two approaches to local linearity of aggregation and related functions (e.g., fuzzy implication functions).
The first approach is linked to linear splines F defined on n-ary unit interval with values in the unit interval, for a dimension n equal or greater than 2. Each linear spline can be characterized by non-overlapping simplexes $E_1, \ldots, E_k$, such that their union is just n-ary unit interval and a restriction of function F/E_i is an affine function on E_i, for $i = 1, \ldots, k$. Note that k is bounded from bellow by n! . We introduce several examples, in particular for case when k = 2. Then necessarily n = 2, and we can consider only two simplexes couples, and the related linear splines are then completely derived by values F(0,0), F(1,0), F(1,0) and F(1,1). We recover several well-known fuzzy logical connectives, such as triangular norms and conorms (Lukasiewicz norm, Lukasiewicz conorm, Min, Max), but also Lukasiewicz implication or Kleene-Dienes implication. Also piece-wise linear copulas are discussed.
Observe that ordinal sums of triangular norms, triangular conorms or copulas preserve the piecewise linearity. For copulas, also W-ordinal sums preserve the piecewise linearity, but not PI-ordinal sums. For preserving the piecewise linearity, also piecewise linear automorphisms of the unit interval could be considered, as well as piecewise linear decreasing bijections (recall, e.g., dual aggregation functions).
The second approach is inspired by the positive homogeneity of aggregation functions, which can be formulated as the linearity of F on any segment <on, xn>, where on=(0, ... , 0) and xn is a point from n-ary unit interval. For fixed point yn from this interval, function F is called an yn-linear function whenever, for any point zn different from yn, the restriction of function F to the related segment, i.e. F/<yn, zn> is linear. Several properties and classes of yn-linear functions are discussed, in particular for the case n=2. As an important example covered by both linearization approaches, the Choquet integral is characterized by an alternative axiomatic approach.

Acknowledgement: The support of the grants VEGA 1/0468/20, VEGA 1/0036/23 and APVV-22-0009 is kindly announced.

References:
[1] Stupňanová A., Su Y., Mesiar R. End-point linear functions. Iranian Journal of Fuzzy Systems 18, (2021), pp. 1-12.
[2] Stupňanová A., Su Y. On some generalizations of homogeneity of aggregation functions. Joint Proceedings of the 19th World Congress of the International Fuzzy Systems Association (IFSA), the 12th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT), and the 11th International Summer School on Aggregation Operators (AGOP). 1. vyd. Dordrecht : Atlantis Press, 2021, pp. 580-583.

Construction methods for triangular norms on bounded trellises

Lemnaouar Zedam[1,2] and Bernard De Baets[1]

[1] KERMIT, Department of Data Analysis and Mathematical Modelling, Ghent University, Coupure links 653, B-9000 Gent, Belgium

[2] LMPA, Department of Mathematics, University of M'sila, 28000 M'sila, Algeria
lemnaouar.zedam@univ-msila.dz, bernard.debaets@ugent.be

Recently [10], we have introduced and given some basic examples of t-norms on a more general mathematical structure known under various names, such as trellises [4], tournament lattices [2] or weakly associative lattices [1,3]. A trellis is more general than a lattice since the partial order relation is replaced by a more general reflexive and antisymmetric relation, while preserving the existence of meets and joins of 2-element subsets. We have provided a generic construction method that allows to extend a t-norm on an interior range of a given meet-semi-trellis to the entire meet-semi-trellis. In this contribution, we discuss more alternative construction methods to obtain t-norms on bounded trellises. We first discuss a construction method based on retractions. Inspired by the fact that ordinal sums are the most important constructions studied in the theory of t-norms on the unit interval [6] and on bounded lattices [1,3,7,8], we pay special attention to this construction method in the setting of trellises.

Keywords: Pseudo-ordered set; trellis; t-norm.

Acknowledgements The first author Lemnaouar Zedam wishes to acknowledge the support provided by: KERMIT, Ghent University, Belgium.

References

1. G.D. Çaylı, Some methods to obtain t-norms and t-conorms on bounded lattices, Kybernetika 55 (2019) 273–294.
2. I. Chajda and J. Niederle, Ideals of weakly associative lattices and pseudo-ordered sets, Archivum Mathematicum 13 (1977) 181–186.
3. A. Dvořák and M. Holčapek, New construction of an ordinal sum of t-norms and t-conorms on bounded lattices, Information Sciences 515 (2020) 116–131.
4. E. Fried, Tournaments and non-associative lattices, Ann. Univ. Sci. Budapest, Sect. Math 13 (1970) 151–164.
5. E. Fried and V.T. Sós, Weakly associative lattices and projective planes, Algebra Universalis 5 (1975) 114–119.
6. E.P. Klement, R. Mesiar and E. Pap, Triangular Norms, Kluwer Academic Publishers, Dordrecht, 2000.
7. Y. Ouyang, H.-P. Zhang and B. De Baets, Ordinal sums of triangular norms on a bounded lattice Fuzzy Sets and Systems 408 (2021) 1–12.
8. Y. Ouyang, H. P. Zhang, Z. Wang and B. De Baets, On triangular norms representable as ordinal sums based on interior operators on a bounded meet semilattice, Fuzzy Sets and Systems 439 (2022) 89–101.
9. H. Skala, Trellis theory, Algebra Universalis 1 (1971) 218–233.
10. L. Zedam and B. De Baets, Triangular norms on bounded trellises, Fuzzy Sets and Systems, https://doi.org/10.1016/j.fss.2023.01.003.

Transformations and truncation of ordinal sums based on the three basic copulas

Susanne Saminger-Platz[1], Anna Kolesárová[2], Adam Šeliga[2], Radko Mesiar[2], and Erich Peter Klement[1]

[1] Institute for Mathematical Methods in Medicine and Data-Based Modeling
Johannes Kepler University Linz, Linz, Austria
{susanne.saminger, ep.klement}@jku.at
[2] Department of Mathematics and Descriptive Geometry, Faculty of Civil Engineering
Slovak University of Technology, Bratislava, Slovakia
{anna.kolesarova, adam.seliga, radko.mesiar}@stuba.sk

Copulas, quasi-copulas and semicopulas have obtained interest as special types of aggregation functions and due to their relevance in theory and applications like, e.g., in probability theory and statistics or applications in finance and hydrology. Different construction methods for such aggregation functions have already been discussed in the past: block-wise methods leading to different types of ordinal sums, grid constructions or patchwork results; perturbation methods transforming copulas into functions sometimes leading to quasi-copulas or even copulas; or considering truncation for ensuring some characteristic properties or necessary conditions for being a (quasi-)copula.

We focus on M-, Π-, and W-ordinal sums with summands from M, Π and W only and their perturbations. The perturbations involve some parameter θ such that, starting from some function F a family of functions (F_θ) is being obtained: monotone increasing with the family parameter θ though possibly losing 2-increasingness, the 1-Lipschitz property or even monotonicity of the original function F itself. As such the question for parameter sets leading to copulas or quasi-copulas is naturally motivated.

Being aware of the results by Hürlimann, who successfully applied truncation by W and M to perturbations of Π, as such leading to an extension of the Eyraud-Farley-Gumble-Morgenstern family of copulas, or the results by De Baets et al. on the truncation of modular functions for obtaining copulas, also an additional truncation by W and M has been investigated leading to larger (sometimes and interestingly even non-convex) sets of parameters ensuring that the obtained functions are copulas. In particular, we shall discuss conditions on the parameters θ for Π-ordinal sums with summands chosen from one of three basic copulas M, Π, W. We will provide examples of such Π-ordinal sums with minimal resp. maximal parameter sets and show that the first and last summand copula play a particular role. We will also present an interesting example of a Π-ordinal sum where truncation helps to largely extend the set of admissible parameters.

Acknowledgements.

Anna Kolesárová, Adam Šeliga and Radko Mesiar gratefully acknowledge the support of the grants VEGA 1/0468/20 and VEGA 1/0036/23.

References

1. De Baets, B., De Meyer, H., Kalická, J., Mesiar, R.: On the relationship between modular functions and copulas. Fuzzy Sets and Systems **268**, 110–126 (2015)
2. Dolati, A., Úbeda-Flores, M.: Constructing copulas by means of pairs of order statistics. Kybernetika **45**, 992–1002 (2009)
3. Hürlimann, W.: A comprehensive extension of the FGM copula. Statist. Papers **58**, 373–392 (2017)
4. Klement, E.P., Kolesárová, A.: Extension to copulas and quasi-copulas as special 1-Lipschitz aggregation operators. Kybernetika **41**, 329–348 (2005)
5. Kolesárová, A., Mesiar, R., Kalická, J.: On a new construction of 1-Lipschitz aggregation functions, quasi-copulas and copulas. Fuzzy Sets and Systems **226**, 19–31 (2013)
6. Manstavičius, M., Bagdonas, G.: A class of bivariate copula mappings. Fuzzy Sets and Systems **354**, 48–62 (2019)
7. Mesiar, R., Komorníková, M., Komorník, J.: Perturbation of bivariate copulas. Fuzzy Sets and Systems **268**, 127–140 (2015)

Monotonicity of binary operations: an unexplored territory

Bernard De Baets[1] and Lemnaouar Zedam[1,2]

[1] KERMIT, Department of Data Analysis and Mathematical Modelling, Ghent University, Coupure links 653, B-9000 Gent, Belgium

[2] LMPA, Department of Mathematics, University of M'sila, 28000 M'sila, Algeria
bernard.debaets@ugent.be, lemnaouar.zedam@univ-msila.dz

The key property of the partial order relation of a poset or lattice is its transitivity. Lack of transitivity can manifest itself in two flavors: the presence of cycles (A better than B, B better than C, and C better than A) or simply incomparability (A better than B, B better than C, but A and C being incomparable). Corresponding mathematical structures are pseudo-ordered sets (psosets, for short) and, in particular, trellises [2,4]. Psosets generalize posets by eliminating the transitivity property, while (proper) trellises do the same compared to lattices, while preserving the existence of meets and joins of 2-element subsets.

The impact of abandoning transitivity causes the meet and join operations of the related proper trellises no longer to be associative (hence, the alternative name 'weakly associative lattices' for trellises [1,3]), and of interest to this contribution, no longer to be increasing. In this note, we aim to investigate weaker forms of monotonicity of binary operations on a trellis and/or a lattice. We restrict our attention to those satisfied by the meet and join operations of a trellis or lattice. Furthermore, we demonstrate the role of these weaker forms in the characterization of the meet and join operations of a trellis, lattice or chain as specific idempotent operations.

Keywords: Binary operation, monotonicity, join, meet, lattice, trellis.

Acknowledgements The second author Lemnaouar Zedam wishes to acknowledge the support provided by: KERMIT, Ghent University, Belgium.

References

1. I. Chajda and J. Niederle, Ideals of weakly associative lattices and pseudo-ordered sets, Archivum Mathematicum 13 (1977) 181–186.
2. E. Fried, Tournaments and non-associative lattices, Ann. Univ. Sci. Budapest, Sect. Math 13 (1970) 151–164.
3. E. Fried and V.T. Sós, Weakly associative lattices and projective planes, Algebra Universalis 5 (1975) 114–119.
4. H. Skala, Trellis theory, Algebra Universalis 1 (1971) 218–233.

On the structure of the sets of binary lattice operations satisfying weaker forms of increasingness

Yuntian Wang[1,2], Lemnaouar Zedam[2,3], Bao Qing Hu[1,4], and Bernard De Baets[2]

[1] School of Mathematics and Statistics, Wuhan University, Wuhan 430072, People's Republic of China
[2] KERMIT, Department of Data Analysis and Mathematical Modelling, Ghent University, Coupure links 653, B-9000 Gent, Belgium
[3] LMPA, Department of Mathematics, University of M'sila, 28000 M'sila, Algeria
[4] School of Mathematics and Physics, Guangxi University of Nationalities, Nanning 530006, People's Republic of China
ytwang.math@whu.edu.cn,lemnaouar.zedam@ugent.be,
bqhu@whu.edu.cn,bernard.debaets@ugent.be

Keywords: Binary operation · increasingness · lattice .

Increasingness w.r.t. each argument is fundamental to the definition of aggregation functions; however, this may exclude certain sound functions such as the mode function or the Lehmer mean from the framework of aggregation functions when fusing data. To address this problem, 'weak monotonicity' [2] and 'directional monotonicity' [4] were proposed as relaxation of the increasingness condition for aggregation functions. These notions have also been extended to bounded chains [5].

In the context of lattices [3], recently a plethora of alternative weaker forms of increasingness of binary operations have been introduced in [7], providing alternative characterizations of the meet and join operations of a chain, lattice or trellis [6].

However, the structure of the set of binary operations satisfying one of these alternative weaker forms has not yet been investigated. In this contribution, we present first results in that direction, in particular the closedness of these sets under appropriate pointwise operations. Next, we address the lattice and group structure using the well-known concept of dominance between binary operations [1].

References

1. C. Alsina, B. Schweizer, M.J. Frank, Associative functions: triangular norms and copulas, World Scientific, 2006.
2. G. Beliakov, T. Calvo, T. Wilkin, Three types of monotonicity of averaging functions, Knowledge-Based Systems 72 (2014) 114–122.
3. G. Birkhoff, Lattice theory, volume 25, American Mathematical Soc., 1940.
4. H. Bustince, J. Fernandez, A. Kolesárová, R. Mesiar, Directional monotonicity of fusion functions, European Journal of Operational Research 244 (2015) 300–308.
5. L. Magdalena, D. Gómez, J. Montero, S. Cubillo, C. Torres, Generalized pre-aggregations, in: International Fuzzy Systems Association World Congress, Springer, pp. 362–370.
6. H. Skala, Trellis theory, Algebra universalis 1 (1971) 218–233.
7. L. Zedam, B. De Baets, Weaker forms of increasingness of binary operations and their role in the characterization of meet and join operations, submitted to Information Sciences.

Exploring Vertex Representation and Cardinality of Aggregation Functions in Honeycomb-based Polygonal Chains

Grzegorz Moś

University of Silesia in Katowice, Bankowa 12, 40-007 Katowice, Poland
grzegorz.mos@us.edu.pl

The honeycomb-based polygonal chain is a well-established concept in computational geometry, with diverse applications in computer graphics, robotics, and architecture. Aggregating these structures is a novel approach that shows promise in various fields, including the analysis of multi-agent systems such as swarms of robots or insects. In this context, aggregation functions are commonly used to calculate the average behavior of the swarm, making new results highly influential in the development of this field.

Traditionally, honeycomb-based polygonal chains are represented by their previous two vertices and a rotation matrix. We present three new forms of representation that utilize different mathematical fields, including trigonometry, modular arithmetic, and combinatorics. Each approach offers unique and intriguing possibilities for approximating the cardinality of honeycomb-based polygonal chains of specific lengths.

We use the new approximation formulas in aggregation functions to define boundary conditions, which are essential to obtain well-defined aggregation functions and further results. Additionally, we estimate the power of the domain and the set of values to directly limit the number of aggregation functions and their practical applications. By understanding the power of the domain and the set of values, we can identify the most suitable aggregation function for a given problem and optimize its performance for a specific application.

Keywords: Aggregation function · Honeycomb · Polygonal chain · Cardinality · Approximation.

References

1. Gągolewski, M.: Data Fusion: Theory, Methods, and Applications. Institute of Computer Science, Polish Academy of Sciences, Warsaw (2015). https://doi.org/10.5281/zenodo.6960306
2. Grabisch, M., Marichal, J.-L., Mesiar, R., Pap, E.: Aggregation Functions, Encyclopedia of Mathematics and its Applications. Cambridge University Press, Cambridge (2009). https://doi.org/10.1017/CBO9781139644150
3. Kennedy, J., Eberhart, R.: Particle swarm optimization. In: Proceedings of ICNN'95 - International Conference on Neural Networks, vol. 4, pp. 1942–1948. (1995). Institute of Electrical and Electronics Engineers, New York. https://doi.org/10.1109/ICNN.1995.488968
4. Moś, G., Honeycomb-Based Polygonal Chains Aggregation Functions. In: Information Processing and Management of Uncertainty in Knowledge-Based Systems, pp. 384–397. Springer International Publishing, Cham (2022). https://doi.org/10.1007/978-3-031-08971-8_32

A model based on multiple one-period possibilistic Markov chains to simulate the tourist flow generated by a cruise ship docked in Palma's port

M.D.M. Bibiloni-Femenias[1,2], José Guerrero[1,2], J.-J. Miñana[1,2], and O. Valero[1,2]

[1] Departament de Ciències Matemàtiques i Informàtica, Universitat de les Illes Balears, Ctra. Valldemossa km. 7.5, 07122, Palma de Mallorca, Spain
[2] Institut d' Investigació Sanitària Illes Balears (IdISBa), Hospital Universitari Son Espases, 07120 Palma, Illes Balears, Spain.
{m.bibiloni, jose.guerrero, jj.minana,o.valero}@uib.es

This article studies how to simulate the tourist flow in a city from the point of view of the task allocation problem [2,5,6]. We assume that all tourists start their tour from a cruise ship docked in the Port of Palma. Then, each tourist decides which places to visit and the order to do it. The tourist making decision process to choose the path depends on a combination of two stimulus, the utility to visit each place and the distance to go there. The utility is a subjective perception modeled by the stars and reviews downloaded from Google "Things to do" in Palma and the distance is computed by the Vincentry distance. Using conjunctive aggregation functions, specifically the minimum and the product, and the harmonic mean as internal aggregation function [4], we merge this information to define the tourist response and, thus, to model how the tourist feels stimulated to visit the different places of the city over the time. The tourist decision making process is modeled by two different decision making methods. One of them solves the celebrated Bellman-Zadeh optimization problem selecting the maximum value computed by the aggregation function [1]. The other one is based on sampling of possibility distributions (in the sense of [3]), without following any optimality criterion. In order to generate the simulations, all the aforementioned ingredients are put in common by means of multiple one-period possibilistic Markov chains which provide the route chosen by tourists over time. The simulations and the model are implemented in Python.

References

1. Bellman, R.E., Zadeh, L.A.: Decision-making in a fuzzy environment. Management science **17**(4), B–141 (1970)
2. Bonabeau, E., Marco, D.d.R.D.F., Dorigo, M., Théraulaz, G., Theraulaz, G., et al.: Swarm intelligence: from natural to artificial systems. No. 1, Oxford university press (1999)
3. Chanas, S., Nowakowski, M.: Single value simulation of fuzzy variable. Fuzzy Sets and Systems **25**(1), 43–57 (1988)
4. Grabisch, M., Marichal, J.L., Mesiar, R., Pap, E.: Aggregation functions. No. 127, Cambridge University Press (2009)
5. Guerrero, J., Miñana, J.J., Valero, O., Oliver, G.: Indistinguishability operators applied to task allocation problems in multi-agent systems. Applied Sciences **7**(10), 963 (2017)
6. Guerrero, J., Valero, Ó., Oliver, G.: Toward a possibilistic swarm multi-robot task allocation: theoretical and experimental results. Neural Processing Letters **46**(3), 881–897 (2017)

Ordinal sum of commutative semigroups on bounded lattices

Martin Kalina

Slovak University of Technology in Bratislava, 810 05 Bratislava, Slovakia martin.kalina@stuba.sk

Keywords: Ordinal sum · Commutative semigroup · Bounded lattice.

In the last twenty years, many papers on ordinal sum of associative monotone operations have been published, see, e. g., [4,5]. In most of them, two kinds of problems are solved. Namely, under which conditions the resulting operation is again associative, and the other problem addressed are various modifications of construction possibilities for ordinal sums yielding associativity. All this study has been initialized by the seminal work [1]. Another paper that started the research, is [2].

In this contribution, we will be focused in presenting another problem. Namely, if the result of an ordinal sum construction is a commutative semigroup, what type of operations we can get as the result. The main type of semigroups we will deal with will be uninorms. An inspiration for this research was teh paper [3] and the references therein.

Acknowledgement

The work on this contribution is supported by the VEGA Grant Agency by grants Nr. 2/0142/20 and 1/0036/23.

References

1. Clifford, A.H.: Naturally totally ordered commutative semigroups. Amer. J. Math. **76**, 631—646 (1954)
2. De Baets, B., Mesiar, R.: Ordinal sums of aggregation operators. In: Bouchon-Meunier, B., Gutiérrez-Ríos, J., Magdalena, L., Yager R.R. (Eds.), Technologies for Constructing Intelligent Systems, vol. 2: Tools, pp. 137–148. Physica-Verlag, Heidelberg (2002).
3. Mesiarová-Zemánková, A.: Characterization of uninorms with continuous uderlying t-norm and t-conorm by means of the ordinal sum construction. International Journal of Approximate Reasoning **83**, 176–192 (2017)
4. Saminger, S.: On ordinal sums of triangular norms on bounded lattices. Fuzzy Sets and Systems **157**, 1403–1116 (2006)
5. Saminger-Platz, S., Klement, E.P., Mesiar, R.: On extensions of triangular norms on bounded lattices. Indag. Mathern.. N.S. **19**(1), 135–150 (2008)

Characterizing discrete (S, N)-implications generated from a non-smooth negation

Marc Munar[1,2], Sebastia Massanet[1,2], and Daniel Ruiz-Aguilera[1,2]

[1] Soft Computing, Image Processing and Aggregation (SCOPIA) research group, University of the Balearic Islands, Palma 07122, Balearic Islands, Spain {marc.munar, s.massanet, daniel.ruiz}@uib.es
[2] Health Research Institute of the Balearic Islands (IdISBa), Palma 07120, Balearic Islands, Spain

While the standard domain for fuzzy logic connectives is the unit interval, the use of finite chains as a representation tool for linguistic labels is more appropriate for the handling of qualitative information. In such cases, the finite chain $L_n = \{0, 1, ..., n\}$ provides a general framework for expressing any finite, totally ordered set and the operators defined on it. Despite the significant research into certain families of operators defined on L_n [1], some families have yet to be thoroughly studied. An example is the family of discrete (S, N)-implications, which remains to be fully characterized. It is worth noting that, in the $[0, 1]$ domain, several efforts have been made to achieve the full characterization of this family. Indeed, it has been characterized in [2] when N is continuous, and in [3] when N is a negation with a single discontinuity point and the t-conorm is the maximum or strict.

In this research, we tackle the characterization of discrete (S, N)-implications, i.e., given a binary operator $I : L_n^2 \to L_n$, we aim to determine the conditions under which there exists a discrete t-conorm S and a discrete negation N such that $I(x, y) = S(N(x), y)$ holds for all $x, y \in L_n$. The general characterization in the discrete case resembles the analogous in the $[0, 1]$ framework. Specifically, it also involves the completion of discrete t-conorms known on a subregion of L_n^2 but without further assumptions on the natural negation N_I, given by $N_I(x) = I(x, 0)$, for all $x \in L_n$. However, this process also inevitably leads to a study of when the underlying discrete pre-t-conorm can be completed to a discrete t-conorm, which diverges significantly from the $[0, 1]$ case. To achieve this, the property of smoothness, which is typically regarded as equivalent to continuity in discrete settings, is considered. The rigid structure of smooth discrete t-conorms, which can be represented as an ordinal sum of Łukasiewicz discrete t-conorms [1], makes it possible to establish sufficient conditions to determine when a smooth discrete pre-t-conorm can be completed to a smooth discrete t-conorm when the natural negation has only one point of non-smoothness. Furthermore, in the cases studied, it has been determined whether the completion is unique or, if not, the number of possible completions.

Acknowledgements This work was partially supported by the R+D+i Project PID2020-113870GB-I00- "Desarrollo de herramientas de Soft Computing para la Ayuda al Diagnóstico Clínico y a la Gestión de Emergencias (HESOCODICE)", funded by MCIN/AEI/10.13039/501100011033/.

References

1. A review on logical connectives defined on finite chains. Fuzzy Sets and Systems (2023), In Press. DOI=10.1016/j.fss.2023.01.004
2. Baczyński, M., Jayaram, B.: On the characterizations of (S, N)-implications. Fuzzy Sets and Systems **158**(15), 1713–1727 (2007)
3. Fernandez-Peralta, R., Massanet, S., Mesiarová-Zemánková, A., Mir, A.: A general framework for the characterization of (S,N)-implications with a non-continuous negation based on completions of t-conorms. Fuzzy Sets and Systems **441**, 1–32 (2022)

Characterization and construction of the continuous completions of some pre-t-norms

Raquel Fernandez-Peralta[1,2], Sebastia Massanet[1,2], Andrea Mesiarová-Zemánková[3,4], and Arnau Mir[1,2]

[1] Soft Computing, Image Processing and Aggregation (SCOPIA) Research Group, Dept. Mathematics and Computer Science, University of the Balearic Islands, 07122 Palma, Spain
{r.fernandez,s.massanet,arnau.mir}@uib.es
[2] Health Research Institute of the Balearic Islands (IdISBa), 07010 Palma, Spain
[3] Mathematical Institute, Slovak Academy of Sciences, 81473 Bratislava, Slovakia zemankova@mat.savba.sk
[4] Institute for Research and Applications of Fuzzy Modelling, University of Ostrava, 70103 Ostrava, Czech Republic

The question of whether a continuous t-norm whose values in a subregion of the unit square are unknown can be (uniquely) completed is a classical and significant problem in the study of these operators [1]. The results regarding this topic are valuable since they disclose important information about their structure, for instance, which subregions of the domain determine the rest of the values uniquely. This problem is usually studied for continuous Archimedean t-norms, since thanks to the existence of additive generators the problem can be interpreted as a functional equation, which variates depending on the unknown region.

In [2] the authors uncover another incentive for the study of this problem. By the duality between t-norms and t-conorms, they prove that the characterization of (S,N)-implications where N is a non-continuous fuzzy negation is equivalent to the problem of the completion of a t-norm whose expression is unknown in a subregion of the unit square, where the unknown region is determined by the discontinuities of the respective fuzzy negation. Particularly, in the case when N has one point of discontinuity, the interest relies on the determination of the continuous completions of pre-t-norms defined in eight specific regions which, up to our knowledge, have not been previously considered in the literature. Thus, our contribution is to provide the continuous completions of cancellative and conditionally cancellative pre-t-norms defined in these eight regions.

The obtained results are very different depending on the region and the cancellative and the conditionally cancellative situations, so several cases have had to be analysed and a specific approach was necessary for almost each case. Depending on the case, the corresponding pre-t-norm can be completed uniquely or it has an infinite number of completions, but in all cases we provide the construction of all the continuous completions in terms of an additive generator.

Acknowledgements

This work was partially supported by the R+D+i Project PID2020-113870GB-I00-"Desarrollo de herramientas de Soft Computing para la Ayuda al Diagnóstico Clínico y a la Gestión de Emergencias (HESOCODICE)", funded by MCIN/AEI/10.13039/501100011033/. Raquel Fernandez-Peralta benefits from the fellowship FPU18/05664 granted by the Spanish Ministry of Science, Innovation and Universities within the Training University Lecturers (FPU) program. Andrea Mesiarová-Zemánková was supported by grant APVV-20-0069.

References

1. Alsina, C., Frank, M., Schweizer, B.: Associative Functions: Triangular Norms and Copulas. World Scientific Publishing Company (2006)
2. Fernandez-Peralta, R., Massanet, S., Mesiarová-Zemánková, A., Mir, A.: A general framework for the characterization of (S,N)-implications with a non-continuous negation based on completions of t-conorms. Fuzzy Sets and Systems **441**, 1–32 (2022)

Part IV

SS1: Interval uncertainty

Necessary and sufficient conditions for differentiability of interval-valued functions

Beatriz Hernández-Jiménez[1], Rafaela Osuna-Gómez[2], Tiago M. Da Costa[3], and Antonio Pascual-Acosta[2]

[1] Universidad Pablo de Olavide, Sevilla, Spain
[2] Universidad de Sevilla, Sevilla, Spain
[3] Universidad de Tarapacá
mbherjim@upo.es, {rafaela,apascual}@us.es, grafunjo@yahoo.es

After a review on differentiability notions for interval-valued functions you can fin in the literature, we present necessary and sufficient conditions for generalized Hukuhara differentiability of interval-valued functions and counterexamples of some equivalences previously presented in the literature, for which important results are based on.

Differential Calculus is a branch of Mathematics that allows us to solve problems where the change of variables can be modeled in a numerical continuum to determine, from it, the variation of these elements in specific moment or interval. The Optimization Theory is a basic part of Applied Mathematics, and the development of differential calculus has enabled powerful mathematical tools for this area.

The Differential Calculus has provided essential mathematical tools to areas as physics, biology, engineering, economics, among others. In particular, since Fermat and Lagrange's work, Differential Calculus has played a leading role in the Optimization Theory. In order to optimize a differentiable function or to solve an optimization problem with constraints, derivative is crucial in both situations and numerical algorithms for computing approximately optimal solutions because the main iterative optimization methods are based on the evaluation of hessian matrices or gradients.

Under the hypothesis that observations and estimates in the real world are incomplete to accurately represent the actual data, the Interval Analysis was introduced by Moore with the aim of managing the imprecision or lack of accurate information that appears on many mathematical models or computational of some real-world deterministic phenomena.

Moreover, interval differentiability and its application in fuzzy environment is an active research area as you can see in literature. But it has not been developed without problems, in order to define correctly the operations between intervals and to establish the appropriate differentiability concept due to the no linearity of the space of intervals. Therefore, it is of interest to establish the definitions and equivalences correctly, such that they allow a successful development of the theory and applications based on them.

So, in this talk we present necessary and sufficient conditions for generalized Hukuhara differentiability of interval-valued functions and counterexamples of some equivalences previously presented in the literature, for which important results are based on.

Keywords: conditions for differentiability · interval-valued functions· generalized Hukuhara differentiability .

Acknowledgements This participation is supported by the European Regional Development Fund (ERDF) and by the Ministry of Economy, Knowledge, Business and University, of the Junta de Andalucía, within the framework of the FEDER Andalucía 2014-2020 operational program. Specific objective 1.2.3. "Promotion and generation of frontier knowledge and knowledge oriented to the challenges of society, development of emerging technologies") within the framework of the reference research project (UPO-1381297). ERDF cofinancing percentage 80%.

References

1. Moore, R.E.: Interval Analysis. Englewood Cliffs NJ. Prince-Hall. (1966).
2. Ghosh, D.: Newton method to obtain efficient solutions of the optimization problems with interval-valued objective functions. J Appl Math Comput. **53**,709–731 (2017).
3. Ghosh, D., Singh, A., Shukla, K.K,, Manchanda, K.: Extended Karush-Kuhn-Tucker condition for constrained interval optimization problems and its application in support vector machines. Inf Sci. **504**: 276–292 (2019).
4. Osuna-Gómez, R., Da Costa, T.M,, Chalco-Cano, Y., Hernández-Jiménez, B.: Quasilinear approximation for interval-valued functions via generalized Hukuhara differentiability. Comp Appl Math. **41**, 149 (2022).

5. Stefanini, L.: A generalization of Hukuhara difference for interval and fuzzy arithmetic. Soft Methods for Handling Variability and Imprecision, Series on Advances in Soft Computing: Springer (2008).
6. Stefanini, L., Bede, B.: Generalized Hukuhara differentiability of interval-valued functions and interval differential equations. Nonlinear Anal. **71**(34),1311–1328 (2009).

Interval-based extensions of Nominal classification method and its application in disease diagnosis

Debashree Guha[1], Soumita Guria[1], and Bapi Dutta[2]

[1] Indian Institute of Technology Kharagpur, Kharagpur 721302, West Bengal, India
[2] Department of Computer Science, University of Jaén, Jaén, Spain
debashree_smst@smst.iitkgp.ac.in, soumitaguriaphd22@kgpian.iitkgp.ac.in
bapi.iitr@gmail.com

Nearly 50% of the world's population (7.4 billion) live in a state of poor access to primary and preventive healthcare. The vast majority of the population in India, particularly in rural areas, do not have access to basic healthcare services. They either do not report illnesses or receive sub-optimum care, both of which result in the accumulation of disease burden. In this scenario implementation of a medical knowledge-enabled "diagnostic decision support" system is a beneficial medical decision-making aid. In medical diagnosis, to assign one of several classes of disease diagnosis, every patient is analyzed depending on her/his symptoms, signs, and medical results. Then we will determine the most appropriate disease class(es) for each patient. This problem can be treated as a multiclass nominal classification problem. The assignment of an alternative (for the above scenario, patients can be treated as alternatives) into a predefined homogenous group (for example, disease class) is known as the classification method. In this study, we present an interval-based extension of CAT-SD (Categorization by Similarity-Dissimilarity), a multicriteria nominal classification (categories are predefined but they have no order between them) method, where the performance value of an action concerning multiple criteria are interval numbers. In a multi-criteria nominal classification method, actions are evaluated depending on multiple criteria, then actions are assigned to nominal categories. Reference actions are used as a representative of the category. Comparing each action with a set of representative actions of the category depending on the likeness threshold, we determine the appropriate category for assigning the actions. To model the uncertainty of medical data, we use interval data for more accurate prediction. Generally, the assignment of actions (e.g., patients) into various categories (e.g., disease classes) is involved into the similarity and dissimilarity with the reference actions (e.g., patients) concerning different criteria (e.g., symptoms such as fever, pain, body weakness, cough, etc). For this purpose, we define the normalized distance between one interval (performance value of action) and other intervals (performance value of reference action) in the following way

$g_{ab} : A_j * B_j \to \mathbb{R}$ such that

$$g_{ab}([a^-, a^+], [b^-, b^+]) = \frac{(b^- - a^+)}{(b^+ - b^-) + (a^+ - a^-)} \tag{1}$$

Where A_j and B_j is the set of performance values of action a and reference action b concerning criteria g_j, where $g_j(a) = [a^-, a^+]$ and $g_j(b) = [b^-, b^+]$ are the performance values with respect to criteria g_j.Next, we define the similarity-dissimilarity function for each criterion among action a and reference action b on interval data based on our proposed function given by equation (1). Finally, we define a likeness function among action a and reference action b depending on the similarity-dissimilarity function. Then comparing likeness degree to respective likeness thresholds for each category, we will assign the actions (e.g., patients) to the most appropriate category (e.g., disease class). The method is established with necessary proofs and it is illustrated in the numerical example in disease diagnosis.

Keywords: Nominal Classification · Interval arithmetic· Normalized distance function .

Acknowledgements

The work is partially supported with grant IIT/SRIC/MM/DV H/2020–2021/143 funded by ISIRD,IIT Kharagpur. The work is also supported by the grant of SERB, Government of India (Ref. No: SPG/2022/00004).

References

1. Costa, A. S., Figueira, J. R., Borbinha, J.: A multiple criteria nominal classification method based on the concepts of similarity and dissimilarity. European Journal of Operational Research.271,193-209(2018)

2. Fernández, E., Figueira, J. R., Navarro, J.: Interval-based extensions of two outranking methods for multi-criteria ordinal classification.Omega Int J Manag Sci.95,102065(2020)
3. Fernández E , Figueira J , Navarro J . An interval extension of the outranking approach and its application to multiple-criteria ordinal classification. Omega Int J Manag Sci.84,189–98(2019)

Nonrepresentable geometric means on interval values sets

Humberto Bustince[1], Pawel Drygaś[2], and Antonio Roldán López de Hierro[3]

[1] Departamento de Estadística, Informática y Matemáticas, Universidad Pública de Navarra, Pamplona
[2] College of natural sciences, University of Rzeszów
[3] Department of Statistics and Operations Research, University of Granada

Keywords: interval value aggregation function, interval value pre-aggregation function, nonrepresentable operation

In this article, we study a special class of operators on interval valued fuzzy sets, which are nonrepresentable. In addition, they take into account the uncertainty interval of the input data. Unfortunately, due to the lack of monotony, directional monotonicity is considered, so that some of these operations are functions of preagregation (see [1]). This is the next step in generalizing the concept of weak operator discussed in the papers [2][3]. But here we propose an operation similar to the representable geometric mean, except that when aggregating we take both ends of the intervals with the appropriate weights. The weights depend here on the measure of uncertainty of the aggregated values. This means that for different arguments the weights may be different. An important advantage of this type of aggregation is the fact that the obtained result has a smaller measure of uncertainty interval than when using standard means.

References

1. H. Bustince, A. Kolesarova, J. Fernandez, R. Mesiar: Directional monotonicity of fusion functions. In: European Journal of Operational Research, 244, (2015) 300-308.
2. P. Drygaś, B. Pekala, K. Balicki and D. Kosior: Influence of new interval-valued pre-aggregation function on medical decision making. In: 2020 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), doi:10.1109/FUZZ48607.2020.9177801.
3. P. Drygaś: On a special class of interval-valued fuzzy operators. In: The 11th International Summer School on Aggregation Operators (AGOP), 2021.

Part V

SS2: Information fusion techniques based on aggregation functions, preaggregation functions and their generalizations

A new approach to select the best method for a fuzzy rule based inference

Asier Urio-Larrea[1], Graçaliz Dimuro[2], Giancarlo Lucca[2], Cedric Marco-Detchart, Francisco Javier Fernandez[1], and Humberto Bustince[1]

[1] Public university of Navarra and Institute of Smart Cities, Spain
[2] Universidade Federal do Rio Grande do Norte, Brazil
asier.urio,bustince,fcojavier.fernandez@unavarra.es, giancarlo.lucca88@gmail.com, gracaliz@gmail.com, cedmarde@upv.es

Keywords: Fuzzy Rules, Generalized Modus Ponens, Rule comparison

The generalized modus ponens, introduced by Zadeh in 1965 [1], has been widely used to create inference systems which deal with imprecise information. This systems use if-then rules, which have fuzzy sets as antecedents and consequents. Zadeh also proposed the compositional rule of inference [2], a mechanism able to resolve the GMP. More methods have been developed ([3][4][5][6]) with the same purpose. For a given application not only the method to be used has to be chosen but also the parameters of it have to be determined ([7][8])

The main goal of this contribution is to establish a rule comparison measure which employs aggregation functions in partciular overlap indices an similarity measures. The secondary goal is to present a decision making system. This system will use the defined comparison measure to select the best method to resolve the GMP.

Acknowledgements

This work has was supported by research project PID2019-108392GB-I00 (AEI/10.13039/501100011033) of the Agencia Estatal de Investigación. Asier Urio-Larrea has a predoctoral grant from Santander-UPNA 2021/22. Graçaliz Pereira is supported by the projects CNPq (301618/2019-4) and FAPERGS (19/2551-0001279-9)

References

[1] L. Zadeh, Fuzzy sets, Information and Control 8 (3) (1965) 338–353.
[2] L. A. Zadeh, Outline of a new approach to the analysis of complex systems and decision processes, IEEE Transactions on Systems, Man, and Cybernetics SMC-3 (1) (1973) 28–44.
[3] L. Zadeh, The role of fuzzy logic in the management of uncertainty in expert systems, Fuzzy Sets and Systems 11 (1) (1983) 199–227.
[4] W. Pedrycz, Applications of fuzzy relational equations for methods of reasoning in presence of fuzzy data, Fuzzy Sets and Systems 16 (2) (1985) 163–175.
[5] L. Kóczy, K. Hirota, Approximate reasoning by linear rule interpolation and general approximation, International Journal of Approximate Reasoning 9 (3) (1993) 197–225.
[6] S. Garcia-Jimenez, H. Bustince, E. Hüllermeier, R. Mesiar, N. R. Pal, A. Pradera, Overlap indices: Construction of and application to interpolative fuzzy systems, IEEE Transactions on Fuzzy Systems 23 (4) (2015) 1259–1273.
[7] A. Konguetsof, N. Mylonas, B. Papadopoulos, Fuzzy reasoning in the investigation of seismic behavior, Mathematical Methods In The Applied Sciences 43 (13, SI) (2020) 7747–7757.
[8] P. Pagouropoulos, C. D. Tzimopoulos, B. K. Papadopoulos, A method for the detection of the most suitable fuzzy implication for data applications, Evolving Systems 11 (3) (2020) 467–477.

On appropriate ordered weighted averages for the aggregation of scores under uncertainty

Josep Freixas

Escola Politècnica Superior d'Enginyeria de Manresa, Av. Bases de Manresa 61-73 08242 Manresa. Universitat Politècnica de Catalunya.

Many decision-making situations require the evaluation of several judges or agents. In a situation where the agents evaluate candidates, the question arises of how best to aggregate evaluations so as to compare the candidates. The aim of this work is to propose a method of aggregating the evaluations of the agents, which has outstanding properties and serve as a potential evaluative tool in many contexts.

Making decisions (in also artificial intelligence) often leads to aggregating preferences or scores on a given set of alternatives. The concept of the ordered weighted averaging operator, was introduced by Yager, [3] and intensively developed in [4] and subsequent works. The development of an appropriate methodology for obtaining the weights is still an issue of great interest. This work is a contribution in the search of appropriate weights for several decision making problems. Formally an ordered weighted average (OWA) operator of dimension n is a mapping $G^n : R^n \to R$ that has an associated collection of weights $w^n = (w_1^n, \ldots, w_n^n)$ lying in the unit interval and summing to one ($w_i^n \geq 0$ for all i and $\sum_{i=1}^{n} w_i^n = 1$) with

$$G^n(x_1, \ldots, x_n) = \sum_{j=1}^{n} w_j^n y_j$$

where y_j is the j-*th* largest element in $\{x_1, \ldots, x_n\}$. A fundamental aspect of this operator is that the re-ordering step, in particular an x_i is not associated with a particular weight w_i^n but rather a weight is associated with a particular ordered position of the x_is. By choosing different w^n one can implement different aggregation operators. The OWA operator is a non-linear operator as a result of the process of determining the y_js. The mean, the median and some trimmed means are examples of OWAs that are frequently used.

In this paper I propose as a solution an OWA that satisfies compelling properties and whose weights are derived from the binomial distribution in discrete version and from the normal distribution in continuous version. It can also be applied in the fuzzy context in which some data coming from the agents is unknown. The proposed OWA operator satisfies, among others, the following properties:

1. *Idempotency*: $G(c, \ldots, c) = c$ for all $c \in \Gamma$, where Γ stands for the set of available inputs.
2. *Boundness*: If $x_1 \leq \cdots \leq x_n$, then $x_1 \leq G(x_1, \ldots, x_n) \leq x_n$.
3. *Monotonicity*: If $x_1 \leq y_1, \ldots, x_n \leq y_n$, then $G(x_1, \ldots, x_n) \leq G(y_1, \ldots, y_n)$.
4. *Strict Monotonicity*: If $x_1 \leq y_1, \ldots, x_n \leq y_n$ and $(x_1, \ldots, x_n) \neq (y_1, \ldots, y_n)$, then $G(x_1, \ldots, x_n) < G(y_1, \ldots, y_n)$.
5. *Symmetry:* $w_i = w_{n+1-i}$ for all $i = 1, 2, \ldots, n$.
6. *Positivity:* $w_i > 0$ for all i.
7. *ID-monotonicity:* $w_i \leq w_j$ if $i < j$ and $j \leq \lceil \frac{n}{2} \rceil$, and $w_i \geq w_j$ if $i < j$ and $\lfloor \frac{n}{2} \rfloor + 1 \geq i$.
8. *Strict ID-monotonicity:* $w_i < w_j$ if $i < j$, and $j \leq \lceil \frac{n}{2} \rceil$ and $w_i > w_j$ if $i < j$ and $\lfloor \frac{n}{2} \rfloor + 1 \geq i$.

The proposed OWA operator is uniquely characterized in [2] by the property of *invariant average reduction*. I review here some of its good properties:

1. it is based on the binomial (and normal) distribution,
2. it is sensitive to an increase or decrease of any agent's score,
3. it is representative of the panel of agents: all agents' scores count,
4. it discriminates very well, ties among candidates are almost avoided,
5. it mostly concentrates the aggregated score in the intermediate agents' scores,
6. it is consistent with its variants so that reversals are almost nonexistent,
7. it has very little dependence on extreme agents' scores,
8. close versions can prevent of some few manipulators or radical agents,
9. it is also useful to evaluate the post agents' reliability,

10. it is transparent, very simple to be understood and computed,
11. it has been shown to be applicable as a tie-breaking system in open tournaments with a limited number of rounds and many participants, see [1],
12. it can be used to compare restaurants or movies according to the costumer's opinions, which may be different in number and in person.

Acknowledgements

This research is part of the I+D+i project PID2019-104987GB-I00 supported by MCIN/AEI/10.13039/501100011033/.

References

1. Freixas, J. The decline of the Buchholz tiebreaker system: a preferable alternative. *In: Nguyen, N.T., Kowalczyk, R., Mercik, J., Motylska-Kuźma, A. (eds) Transactions on Computational Collective Intelligence XXXVII, 1-20. Lecture Notes in Computer Science, vol 13750. Springer, Berlin, Heidelberg*, 2022.
2. Freixas, J. An aggregation rule based on the binomial distribution. *Mathematics*, 10(23):4418, 2022.
3. Yager, R.R. On ordered weighted averaging aggregation operators in multicriteria decision making. *IEEE Transactions on Systems Man and Cybernetics*, 18:183–190, 1988.
4. Yager, R.R. and Kacprzyk, J. The ordered weighted averaging operators, theory and applications, *Kluwer Academic Publishers*, 1997.

Application of $[a, b]$-aggregation functions in the problem of microarrays regression ensembling

Jan G. Bazan[1], Stanislawa Bazan-Socha[2], Urszula Bentkowska[1], Wojciech Gałka[1], Marcin Mrukowicz[1], and Marcin Wielgos[1]

[1] Institute of Computer Science, University of Rzeszów, Rzeszów, Poland
{jbazan,ubentkowska,wgalka,mmrukowicz}@ur.edu.pl, hello.wielgos@gmail.com
[2] Department of Internal Medicine, Jagiellonian University Medical College Kraków, Poland
stanislawa.bazan-socha@uj.edu.pl

Different classes of aggregation functions defined on arbitrary [a,b] interval, called [a,b]-aggregation functions (cf. [1,2,7]) are examined both from the theoretical and application point of view. The usefulness of [a,b]-aggregation functions is examined in terms of optimizing the performance of the algorithm dedicated to datasets with large number of features (for example microarrays) and continous type of decision. The presented model uses the method of feature selection and regression ensembling (cf. [4,5]). Aggregation functions are used here to combine the output values of the constituent regression models. The proposed model is compared with the bagging regression model with the optimized parameters based on Grid Search. Typical measures such as MSE or RMSE are applied to evaluate the proposed ensemble model. The proposed ensemble regression model with adequate set of parameters (cf. [3,6]) outperforms significantly the corresponding single models (such as for example Support Vector Regression) which is proved using statistical tests.

References

1. Asmus, T., Pereira Dimuro, G., Bedregal, B., Sanz, J. A., Fernandez, J., RodriguezMartinez, I., Mesiar, R., Bustince, H.: A constructive framework to define Fusion functions with floating domains in arbitrary closed real intervals. Information Sciences **610**, 800-829 (2022)
2. Asmus, T., Dimuro, G., Bedregal, B., Rodríguez-Martínes, I., Fernandez, J., Bustince, H.: Negations and dual aggregation functions on arbitrary closed real intervals. In: 2022 IEEE International Conference on Fuzzy Systems, pp. 1-8, FUZZ-IEEE, Padova (2022) doi: 10.1109/FUZZ-IEEE55066.2022.9882708.
3. Costa, V.S., Farias, A.D.S., Bedregal, B., Santiago, R.H.N., de P. Canuto, A.M.: Combining Multiple Algorithms in Classifier Ensembles using Generalized Mixture Functions. Neurocomputing **313**, 402-414 (2018)
4. González, S., García, S., Del Ser, J., Rokach, L., Herrera, F.: A practical tutorial on bagging and boosting based ensembles for machine learning: algorithms, software tools, performance study, practical perspectives and opportunities. Information Fusion **64**, 205-237 (2020)
5. Mendes-Moreira, J., Soares, C., Jorge, A. M., Freire De Sousa, J.: Ensemble approaches for regression: A survey. ACM Computing Surveys **45** (1), Article No. 10, 1-40 (2012)
6. Mohammed, A.M., Onieva, E., Woźniak, M., Martínez-Muñoz, G.: An analysis of heuristic metrics for classifier ensemble pruning based on ordered aggregation. Pattern Recognition **124**, 108493 (2022)
7. Špirková, J., Bustince, H., Fernandez, J., Sesma-Sara, M.: New Classes of the Moderate Deviation Functions. Joint Proceedings of the 19th World Congress of the International Fuzzy Systems Association (IFSA), the 12th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT), and the 11th International Summer School on Aggregation Operators (AGOP), pp. 661-666, Atlantis Studies in Uncertainty Modelling, vol. 3 (2022)

On k-Lipschitzian pseudo-overlap and pseudo-grouping functions

Anderson Cruz[1,3], Rui Paiva[2,4], Helida Santos[2,5], Regivan Santiago[1], Benjamín Bedregal[1], Antonio F. Roldán López de Hierro[6], Javier Fernandez[2,3], and Humberto Bustince[2,3]

[1] Universidade Federal do Rio Grande do Norte, Natal, 59072-970, Brazil anderson@imd.ufrn.br, {regivan, bedregal}@dimap.ufrn.br
[2] Universidad Pública de Navarra, Pamplona, 31006, Spain
[3] Navarra Artificial Intelligence Research (NAIR) Center, Pamplona, 31006, Spain
{fcojavier.fernandez,bustince}@unavarra.es
[4] Instituto Federal de Educação, Ciência e Tecnologia do Ceará, Maracanau, 61936-000, Brazil
rui.brasileiro@ifce.edu.br
[5] Universidade Federal do Rio Grande, Rio Grande, 96203-900, Brazil
helida@furg.br
[6] Universidad de Granada, Granada, 18010, Spain
aroldan@ugr.es

Keywords: Pesudo-overlap · Pseudo-grouping · Lipschitziniaty.

Recently, various extensions of overlap and grouping functions were investigated. Among them, [1.] and [2.], independently, defined the pseudo-overlap and pseudo-grouping functions. They are very useful when the order of the objects being compared is relevant, e.g. in multicriteria decision-making problems and time series analysis. Additionally, k-Lipschitzianity is a very useful condition that promotes the stability of a mathematical model from a practical point of view. This property prevents that a small input distance does not result in a great output distance. Therefore, in this work, we show some new construction methods for pseudo-overlap and pseudo-grouping functions and demonstrate when they satisfy the k-Lipschitz condition.

Acknowledgments

This work was supported by the Brazilian funding agency CNPq under Projects 311429/2020-3 and 200282/2022-0, by the project PID2019-108392GB-I00 (AEI/10.13039/ 501100011033) of the Spanish Government and Fundación "la Caixa".

References

1.. Batista, T., 2022. Generalizações da Integral de Choquet como Método de Combinação em Comitês de Classificadores. Ph.D. thesis. Universidade Federal do Rio Grande do Norte.
2.. Zhang, X., Liang, R., Bustince, H., Bedregal, B., Fernandez, J., Li, M., Ou, Q., 2022. Pseudo overlap functions, fuzzy implications and pseudo grouping functions with applications. Axioms 11, 1–29.

Discrete gradient computation using moderate deviation functions

Carlos Lopez-Molina[1], Mikel Ferrero-Jaurrieta[1], Jana Špirková[2], Marisol Gomez[1], and Humberto Bustince[1]

[1] Universidad Pública de Navarra, 31006 Pamplona, Spain
[2] Laboratory of Artificial Intelligence, Navarrabiomed, Hospital Universitario de Navarra (HUN), 31008 Pamplona, Spain
[3] Matej Bel University Tajovského 10, 975 90 Banská Bystrica, Slovakia
carlos.lopez@unavarra.es, jana.spirkova@umb.sk

The study of convolution kernels for gradient computation has been rather still in the history of image processing. Despite remarkable works have been presented, most of the recent proposals focus on introducing enhanced constraints or properties leading to redefinitions of the concept of optimality and, consequently, producing new convolution kernels. However, it has always been unclear which is the best-performing kernels, or even which is the best possible parameterization of a given family of kernels for a specific problem.

One interesting question then arises, related to the very nature of convolution kernels for image processing. As part of the inheritance from analogical (hence, continuous) signal processing, early attempts to extract features in image processing were based on convolution kernels, which mimicked the existing filters in analogical technologies. While this boosted the early developments in image processing, recent developments question whether other types of natively-digital operators, namely bilateral kernels, are more adequate for the task. Bilateral kernels perform independent evaluation of the tonal and spatial information around each pixel in the image, hence allowing for a more sensible adaptation to different regions of an image. Initially proposed by Tomasi and Manduchi [3], these filters have become a standard for content-aware smoothing (CAS, a.k.a. non-destructive smoothing). Since CAS is a zero-th order operation on a signal, we understand that similar principles could be used for first order differentiation, which is the basis for gradient extraction.

In this work we propose a generalization of the well-known bilateral kernels [2,1] to image differentiation. This proposal focuses on abilitating an independent tonal difference term, based on moderate deviation functions. While the very shape of the convolution kernels accounts for the spatial information, the moderate deviation functions perform the interpretation of the tonal differences. Hence, we can build up a family of first order bilateral kernels that can be configured to fit local spatio-tonal characteristics at each region of an image, or at different images in a dataset.

Acknowledgements

This work has been funded by the Spanish ministry MCIN, with the project PID2019-108392GB-I00/AEI/10.13039/501100011033.

References

1. Danny Barash. A fundamental relationship between bilateral filtering, adaptive smoothing, and the nonlinear diffusion equation. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 24:844–847, 2002.
2. M. Elad. On the origin of the bilateral filter and ways to improve it. *IEEE Trans. on Image Processing*, 11(10):1141–1151, 2002.
3. C. Tomasi and R. Manduchi. Bilateral filtering for gray and color images. In *Proc. of the IEEE International Conf. on Computer Vision*, pages 838–846, 1998.

A first approach to deal with computable aggregations over random variables

Juan Baz[1], Irene Díaz[2], Luis Garmendia[3], Daniel Gómez[4], Luis Magdalena[5], and Susana Montes[1]

[1] Department of Statistics, Operational Research and Didactic of Mathematics, University of Oviedo, Spain
{bazjuan,montes}@uniovi.es
[2] Department Computer Science, University of Oviedo, Spain sirene@uniovi.es
[3] Department of Computer Science, Complutense University of Madrid, Spain lgarmend@fdi.ucm.es
[4] Department of Statistics and Operational Research, Complutense University of Madrid, Spain dagomez@estad.ucm.es
[5] E.T.S. Ingenieros Informáticos, Universidad Politécnica Madrid, Spain luis.magdalena@upm.es

Keywords: Aggregation · Computable aggregation · Probability

Aggregation is one of the hottest disciplines in soft computing. In general, the aggregation process is modelled in terms of an aggregation function, i.e., a mapping from the input (a set of values) into a summarizing aggregated value. In this work we are going to avoid these two assumptions since we will aggregate random variables (instead of values) by means of programs (instead of mathematical functions). This will be the result of merging two concepts previously introduced.

One one hand, computable aggregations were introduced to replace the mathematical function defining the aggregation, by a program that performs the aggregation process [4] There are different reasons to justify this extension. First at all, it is possible to deal with more complex and realistic aggregation process that can't be represented by means of mathematical functions. Secondly, modeling an aggregation process by means of its implementation allows us to explore some computational properties not directly related to the aggregation itself but to its implementation (recursivity, complexity, parallelisation, etc) [2,3].

On the other hand, aggregation functions over random variables were defined with the idea of modeling at least those situations in which the information to be aggregated is obtained as a measurement process over a sample population. In this framework, the aggregation function can be seen as a function that given a vector/set of random variables, returns a random variable as the result of the aggregation process. In that framework, questions as monotonicity and boundary conditions were extended from vectors to random vectors [1]. Also, the concept of monotonicity based on orders between random vectors and random variables was considered. Nevertheless, this aggregation processes were defined based on mathematical functions with some limitations.

In this work we propose a first approach that generalizes both concepts by defining computable aggregations over random variables from different perspectives. Some of the approaches to aggregate random variables are based on their density functions, their distribution functions, their simulation functions or on the concept of random variable itself. To force monotonicity on aggregation processes some concepts as stochastic and probabilistic orders should be considered.

Acknowledgements

J. Baz is supported by Programa Severo Ochoa of Principality of Asturias (BP21042). L. Garmendia and D. Gómez are supported by Goverment of Spain (grant PID2021-122905NB-C21). L. Magdalena is supported by Government of Spain (grants PID2020-112502RB-C41 and PID2021-122905NB-C22), and Comunidad de Madrid (Convenio Plurianual con la UPM en la lınea de actuación Programa de Excelencia para el Profesorado Universitario).

References

1. Juan Baz, Irene Díaz, and Susana Montes. The choice of an appropriate stochastic order to aggregate random variables. In *Building Bridges between Soft and Statistical Methodologies for Data Science*, pages 40–47. Springer, 2022.
2. Luis Magdalena, Luis Garmendia, Daniel Gómez, Ramón González del Campo, Juan Tinguaro Rodríguez, and Javier Montero. Types of recursive computable aggregations. In *2019 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, pages 1–6. IEEE, 2019.

3. Luis Magdalena, Luis Garmendia, Daniel Gómez, and Javier Montero. Hierarchical computable aggregations. In *2022 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, pages 1–8. IEEE, 2022.
4. Javier Montero, Ramón González-del Campo, Luis Garmendia, Daniel Gómez, and J Tinguaro Rodríguez. Computable aggregations. *Information Sciences*, 460:439–449, 2018.

On the preservation of properties when aggregating random vectors and stochastic processes

Juan Baz[1], Irene Díaz[2], and Susana Montes[1]

[1] Department of Statistics, Operational Research and Didactic of Mathematics, University of Oviedo, Spain
{bazjuan,montes}@uniovi.es
[2] Department Computer Science, University of Oviedo, Spain sirene@uniovi.es

Keywords: Aggregation · Stochastic Orders · Stochastic Processes

The concept of aggregation of random variables generalizes the concept of aggregation of real numbers and allows to work with aggregation concepts from a probabilistic approach [1]. Given a probability space, an aggregation of random variables takes a random vector and returns a random variable. The classical conditions of monotonicity and boundary conditions are redefined using stochastic orders, see [5]. The usual stochastic order, also known as Strong Stochastic Dominance, has been proved to be an adequate stochastic order to consider, since it allows the composition of usual aggregation functions with random vectors to be aggregations of random variables [1]. This type of aggregations of random variables are known as induced aggregations of random variables. In this talk, we aim to extend this concept to random vectors and stochastic process and study the preservation of some of its properties in the aggregation process.

Firstly, we define the aggregation of random vectors with respect a stochastic order. In this case, several random vectors, which can be seen as a random matrix, are aggregated to obtain an output random vector. The Strong Stochastic Dominance is preserved when composing random matrices with aggregations of vectors. The definition of other stochastic orders for the case of admissible orders of vectors, see [1], is also studied. In addition, we give sufficient conditions for the preservation of several properties of random vectors as independence of components, exchangeability or zonoid equivalency [3], among others.

Secondly, we generalize again the concept in order to aggregate several stochastic processes to obtain a new stochastic process, all defined over the same index set. The boundary and monotonicity conditions are generalized using stochastic orders for stochastic processes, which usually only care about finite-dimensional distributions. The conditions for which important properties such that Markov, stationary, ergodic or being a martingale, see [4], are preserved are studied. In general, we need the aggregation of stochastic processes to be induced and local, i.e., it can be decomposed in an induced aggregation of random variables for any element of the index set.

Acknowledgements

J. Baz is supported by Programa Severo Ochoa of Principality of Asturias (BP21042).

References

1. J. Baz, I. Díaz, and S. Montes. The choice of an appropriate stochastic order to aggregate random variables. In *Building Bridges between Soft and Statistical Methodologies for Data Science*, pages 40–47. Springer, 2022.
2. H. Bustince, J. Fernández, A. Kolesárová, and R. Mesiar. Generation of linear orders for intervals by means of aggregation functions. *Fuzzy Sets and Systems*, 220:69–77, 2013.
3. I. Molchanov, M. Schmutz, and K. Stucki. Invariance properties of random vectors and stochastic processes based on the zonoid concept. *Bernoulli: a journal of mathematical statistics and probability*, 20(3):1210–1233, 2014.
4. S. M. Ross. *Stochastic processes*. John Wiley & Sons, 1995.
5. M. Shaked and J. G. Shanthikumar. *Stochastic orders*. Springer, 2007.

Improving the performance of a fuzzy rule-based classifier when tackling imbalanced classification problems by applying aggregation and pre-aggregation functions

J. Sanz[1,2], M. Sesma-Sara[1,2], R. Pascual[1,2], and H. Zia[1,2]

[1] Department of Statistics, Computer Science and Mathematics, Universidad Publica de Navarra, Campus Arrosadia s/n, Pamplona, Navarra 31006, Spain joseantonio.sanz@unavarra.es
[2] Institute of Smart Cities, Universidad Publica de Navarra, Campus Arrosadia s/n, Pamplona, Navarra 31006, Spain

Binary imbalanced classification problems are a type of binary classification problems where the class distribution is uneven, that is, the number of examples on one class (the positive class) excels that of the other class (the negative class). Methodologies to tackle theses problems can be embraced in four categories: data preprocessing methods, algorithm modification methods, cost-sensitive methods and ensembles approaches. Last year it was developed a new fuzzy rule-based classifier (named FARCI) [1], which belongs to the second category, on the basis of FARC-HD, which is a state-of-the-art fuzzy classifier to tackle standard classification problems. Specifically, all the learning stages of FARC-HD were modified in order to tackle directly imbalanced classification problems. One of the modifications consists of replacing the product t-norm by other averaging aggregation functions to model the conjunction among fuzzy sets in order to avoid the effect of having rules composed of a different number of conditions in their antecedent part. This new method applies the additive combination fuzzy reasoning method, that is, it considers the usage of the normalized sum, which is a non-averaging aggregation function, to fuse the information given by all the fired rules when classifying new examples. In other words, this inference method sums the association degrees of all the fired rules for each class and it finally predicts the class associated with the largest aggregated value. However, according to the results of the best configuration of the FARCI reported on the paper, the number of learned fuzzy rules belonging to the positive class is larger (almost double) than that of fuzzy rules labeled with the negative class. This fact may provoke that applying the sum could not be fair for both classes because, in general, there could be a larger number of fired fuzzy rules of the positive class, that is, a larger number of elements to be aggregated. This fact could imply that the resulting aggregated value can be larger for the positive class even in situations where the aggregated values are less than those of the negative one. Having these facts in mind, in this work we propose to use different aggregation and pre-aggregation functions [2,3] to fuse the information given by the fired fuzzy rules to try improve the performance of such classifier. Specifically, we apply the maximum (fuzzy reasoning method of the winning rule), the probabilistic sum as well as some generalizations of the Choquet integral, where there are as many fuzzy measures as classes. These fuzzy measures allow one to deal with the problem of having different number of elements to be aggregated because of the construction method considered to assign the weights that will be applied to the elements to be aggregated. The obtained results show that the usage of the generalizations of the Choquet integral allows one to obtain competitive results that, in some cases, even enhance the results provided be the classical normalized sum.

Keywords: Imbalanced classification problems · Imbalanced classification problems, fuzzy rule-based classification systems, aggregation functions, pre-aggregation functions · Aggregation functions · pre-aggregation functions.

Acknowledgements Supported by the Spanish Ministry of Science and Technology (PID2019-108392GB I00 (AEI/10.13039/501100011033)) and the Public University of Navarre, Spain under the project PJUPNA25-2022.

References

1. Sanz, J., Sesma-Sara, M., Bustince, H.: A fuzzy association rule-based classifier for imbalanced classification problems. Information Sciences **577**, 265–279 (2021)
2. Dimuro, G., Fernández, J., Bedregal, B., Mesiar, R., Sanz, J., Lucca, G., Bustince, H.: The state-of-art of the generalizations of the Choquet integral: From aggregation and pre-aggregation to ordered directionally monotone functions. Information Fusion **57**, 27–43 (2020)

3. Lucca, G., Dimuro, G., Fernández, J., Bustince, H., Bedregal, B., Sanz, J.: Improving the Performance of Fuzzy Rule-Based Classification Systems Based on a Nonaveraging Generalization of CC-Integrals Named CF1F2 -Integrals. IEEE Transactions on Fuzzy Systems **27**(1), 124–134 (2019)

Distance transformations applied to membership degrees in fuzzy sets

Laura De Miguel[1,2], Xabier Gonzalez-Garcia[1], Asier Urio-Larrea[1,2], Carlos Lopez-Molina[1], and Humberto Bustince[1,2]

[1] Universidad Pública de Navarra, Campus Arrosadia s/n 31006 Pamplona Spain
[2] Institute of Smart Cities, Campus Arrosadia s/n 31006 Pamplona Spain
laura.demiguel@unavarra.es, asier.urio@unavarra.es, carlos.lopez@unavarra.es, humberto.bustince@unavarra.es

The distance transform (DT) (also known as distance map) is a fundamental tool of mathematical morphology [4]. Their aim is to compute the distance from a point to an object.

In this work we study how the distance transform defined over the unit interval can be satisfactory linked to any fuzzy set or generalization in which the membership degree is represented with a subset in the unit interval [3]. Using the distance transform we define a function that is used to fuse information when different classes of fuzzy sets and their generalization are considered simultaneously. We study the theoretical properties of the function showing an in-depth relation with the properties similarity measures, distance measures and entropy measures of fuzzy sets [1,2].

Acknowledgements

This work has been funded by the Spanish ministry MCIN, with the project PID2019-108392GB-I00/AEI/10.13039/501100011033 and Asier Urio-Larrea has a predoctoral grant from Santander-UPNA 2021/22.

References

1. H Bustince, E Barrenechea, and Miguel Pagola. Image thresholding using restricted equivalence functions and maximizing the measures of similarity. *Fuzzy Sets and Systems*, 158(5):496–516, 2007.
2. H Bustince, E Barrenechea, and Miguel Pagola. Relationship between restricted dissimilarity functions, restricted equivalence functions and normal en-functions: Image thresholding invariant. *Pattern Recognition Letters*, 29(4):525–536, 2008.
3. Humberto Bustince, Edurne Barrenechea, Miguel Pagola, Javier Fernandez, Zeshui Xu, Benjamin Bedregal, Javier Montero, Hani Hagras, Francisco Herrera, and Bernard De Baets. A historical account of types of fuzzy sets and their relationships. *IEEE Transactions on Fuzzy Systems*, 24(1):179–194, 2015.
4. Laurent Najman and Hugues Talbot. *Mathematical morphology: from theory to applications*. John Wiley & Sons, 2013.

Fusion of LiDAR and RGB images for tree detection*

P. Flores-Vidal[1], D. Gómez[1], J. T. Rodríguez[2], and J. Montero[2]

[1] Faculty of Statistical Studies, Complutense University, Madrid, Spain pflores@ucm.es,dagomez@estad.ucm.es

[2] Faculty of Mathematics, Complutense University, Madrid, Spain jtrodrig@ucm.es,monty@mat.ucm.es

In recent years, the use of remote sensing technology has become increasingly popular for mapping and monitoring forested areas. In this research, we introduce a novel approach for detecting trees from a combination of LiDAR (Light Detection and Ranging) and RGB images. Our approach employs OWA aggregation operators to fusion these images to effectively distinguish trees (foreground) from non-trees (background). The fusioned image is processed using different techniques: Image segmentation, mathematical morphology, feature extraction, and object classification. The LiDAR data provides a high resolution of the 3D structure of the terrain, while the RGB image contains other contextual information such as texture, color and edges. Our method takes advantage of these complementary sources of information by fusing them together to improve the accuracy of tree detection. Our proposal is evaluated on a dataset and compared with other state-of-the-art tree detection methods. Results show that in terms of accuracy the proposed method performs at least at the same level than other existing methods . The findings show potential applications in forestry management and environmental monitoring.

Keywords: tree detection, remote sensing, LiDAR, aggregation operator, information fusion, RGB.

Acknowledgments

This research has been partially supported by the Spanish Ministry of Science, grant PGC2018-096509-BI00.
Data was collected with the help of Prof. G. Biging, Department of Environmental Science, Policy, and Management, UC Berkeley, USA.

References

1. Axelsson, P.: DEM generation from laser scanner data using adaptive TIN models. International archives of photogrammetry and remote sensing, **33(4)**, 110–117 (2000)
2. Campbell, J. B., Wynne, R. H.: Introduction to remote sensing. The Guilford Press, New York (2011)
3. Dong, P., Chen, Q.: LiDAR remote sensing and applications. CRC Press, New York (2017)
4. Flores Vidal, P. A.: Problemas de tratamiento de imágenes basados en la incorporación de características humanas. (2019)
5. Ke, Y., Quackenbush, L. J.: A review of methods for automatic individual tree-crown detection and delineation from passive remote sensing. International Journal of Remote Sensing, **32(17)**, pp. 4725–4747. (2011)
6. Xia, J., Yokoya, N., Iwasaki, A.: Fusion of hyperspectral and LiDAR data with a novel ensemble classifier. IEEE Geoscience and Remote Sensing Letters, 15(6), pp. 957–961. (2018)

* Supported by the Spanish Ministry of Science.

Adjusting the Sugeno-like FG-functional concept and its application to fuzzy-rule based classification systems

Giancarlo Lucca[1], Jonata Wieczynski[2], Cedric Marco-Detchart[3], Tiago da Cruz Asmus[4], Heloisa de Arruda Camargo[5], Helida Salles Santos[1], Eduardo Borges[1], Humberto Bustince[2], and G. Dimuro[1]

[1] Centro de Ciências Computacionais, Universidade Federal do Rio Grande, Rio Grande, Brazil
{giancarlo.lucca, helida, eduardoborges, gracalizdimuro}@furg.br
[2] Dep. Estadistica, Informatica y Matem., Universidad Publica de Navarra, Pamplona, Spain
{jonata.wieczynski, bustince}@unavarra.es
[3] Valencian Research Inst. for Artif. Intel., Universitat Politècnica de València, Valencia, Spain
cedmarde@upv.es
[4] Inst. Matem., Estadística e Física, Universidade Federal do Rio Grande, Rio Grande, Brazil
tiagoasmus@furg.br
[5] Departamento de Computação, Universidade Federal de São Carlos, São Carlos, Brazil,
heloisa@dc.ufscar.br

The Sugeno integral [7] is an aggregation function [4] that has been applied in different research fields. For example, this function, when defined in the unit interval [0,1], has been applied in the fuzzy reasoning method (FRM) of a fuzzy rule-based classification system (FRBCS) to aggregate the information related with the different classes of the problem, showing that this is a powerful tool for coping with aggregation processes in classification problems. See [2,3,6] for interesting applications to motor-imagery-based brain computer interfaces and social network analysis.

Recently, the notion of Sugeno-like FG-functional was proposed by Bardozzo et al. [1], as a generalization of the Sugeno Integral on [0,1] to the positive reals by replacing the minimum and the maximum of the definition of the Sugeno integral by functions F and G, respectively, defined also on the positive reals.

However, there are some drawbacks in the definition of the FG-functional, such as the possibility of disregarding the magnitude of the aggregated inputs, since the FG-functionals operate with inputs from the positive reals, but only admit fuzzy measures ranging in the unit interval [0,1]. Then, depending on the chosen F and G, and the magnitude of the inputs (for example, when the inputs are larger than 1), those inputs may be totally ignored in the calculus, with the result considering just the values of the fuzzy measure.

Then, this paper has a two-fold objective. First, we adjust the definition of the FG-functional, adapting the signature of the fuzzy measure to its original definition in the literature [4,5] (that is, in the positive reals), and, consequently, the signatures of the functions F, G and the FG-functional itself, so that it allows inputs larger than 1 without mischaracterizing their role in fuzzy integrals. This new definition is then called SFG-functionals. We also study some basic properties of SFG-functionals, in particular, with some choices of F and G that produce good results in practical problems.

Then, we define several SFG-functionals by adopting different functions as F and G, in order to apply them in the FRM of a FRBCS, where also different fuzzy measures are used. We show that the newly defined SFG-functionals provide competitive results, analyzing the best combinations of F and G functions for this application.

Acknowledgments.

This work was supported by FAPERGS/Brazil (Proc. 19/2551-0001660-3), CNPq/Brazil (Proc. 301618/ 2019-4, 305805/ 2021-5), CNPq/FAPERGS (Edital 07/2022 - Program to Support the Settlement of Young Doctors in Brazil), FAPESP/Brazil (Proc. 2022/ 09136-1), Agencia Estatal de Investigación of Spain, project PID2019-108392GB-I00 (AEI/ 10.13039/ 501100011033), Consellería d'Innovació, Universitats, Ciencia i Societat Digital from Comunitat Valenciana (APOSTD/2021/227), through the European Social Fund (Investing In Your Future), grant from the Research Services of Universitat Politècnica de València (PAID-PD-22), Fundación "La Caixa" (grant LCF/PR/ PR13/51080004), and Navarra de Servicios y Tecnologías.

References

1. Bardozzo, F., De La Osa, B., Horanská, L., Fumanal-Idocin, J., delli Priscoli, M., Troiano, L., Tagliaferri, R., Fernandez, J., Bustince, H.: Sugeno integral generalization applied to improve adaptive image binarization. Information Fusion **68**, 37–45 (2021). https://doi.org/10.1016/j.inffus.2020.10.020

2. Fumanal-Idocin, J., Takáč, Z., Horanská, L., da Cruz Asmus, T., Dimuro, G., Vidaurre, C., Fernandez, J., Bustince, H.: A generalization of the sugeno integral to aggregate interval-valued data: An application to brain computer interface and social network analysis. Fuzzy Sets and Systems **451**, 320–341 (2022). https://doi.org/https://doi.org/10.1016/j.fss.2022.10.003, recent Trends in Aggregation - In Honour of Radko Mesiar's 70th Birthday
3. Fumanal-Idocin, J., Wang, Y.K., Lin, C.T., Fernandez, J., Sanz, J.A., Bustince, H.: Motor-imagery-based brain-computer interface using signal derivation and aggregation functions. IEEE Transactions on Cybernetics **52**(8), 7944–7955 (2022). https://doi.org/10.1109/TCYB.2021.3073210
4. Grabisch, M., Marichal, J., Mesiar, R., Pap, E.: Aggregation Functions. Cambridge University Press, Cambridge (2009)
5. Grabisch, M.: Set Functions, Games and Capacities in Decision Making, Theory and Decision Library C, vol. 46. Springer, Cham (2016)
6. Ko, L.W., Lu, Y.C., Bustince, H., Chang, Y.C., Chang, Y., Ferandez, J., Wang, Y.K., Sanz, J.A., Pereira Dimuro, G., Lin, C.T.: Multimodal fuzzy fusion for enhancing the motor-imagery-based brain computer interface. IEEE Computational Intelligence Magazine **14**(1), 96–106 (2019). https://doi.org/10.1109/MCI.2018.2881647
7. Sugeno, M.: Theory of fuzzy integrals and its applications. Ph.D. thesis, Tokyo Institute of Technology (1974)

Generalization of the ML TSK FS model based on the Choquet integral for Multi-label Classification

Karina Condori[1], Julian Suarez[2], Giancarlo Lucca[3], Qiongdan Lou[4], Zhaohong Deng[4], Tiago C. Asmus[2], Leonardo Emmendorfer[5], Humberto Bustince[6], and Graçaliz P. Dimuro[3]

[1] Programa de Pós-Graduação em Modelagem Computacional, Universidade Federal do Rio Grande, Rio Grande, Brazil
[2] IMEF, Universidade Federal do Rio Grande, Rio Grande, Brazil
[3] C3, Universidade Federal do Rio Grande, Rio Grande, Brazil
[4] Jiangnan University, China
[5] Universidade Federal de Santa Maria, Brazil
[6] Universidad Publica de Navarra Spain

Machine learning techniques reached considerable relevance recently. Automatic classification of all types of digital information, including texts, photos, music, and videos, is in increasing demand. Multi-label classification models based on deep learning are tools that fit naturally to the problems mentioned above. In particular, the Takagi Sugeno Kang Multi-Label Classification model (ML-TSK-FS), proposed by Lou et al. [5], is based on fuzzy inference rules that consider the relationship between features and their labels.

A key component in any Fuzzy Reasoning Method is how to aggregate the information given by the rules triggered during the inference process. In recent years, the aggregation functions based on the Choquet integral [2] and its generalizations (see, e.g.:[1,3,4,7,8]) have shown to be very promising, obtaining improvements in the performance of existing models in the literature, mainly in classification.

For this reason, this work aims to propose a generalization of the ML-TSK-FS model using aggregation functions and the Choquet integral, to obtain a new model for Multi-Label classification, called multi-label Takagi Sugeno Kang Choquet Fuzzy System (ML-TSKC FS).

Observe that the Takagi-Sugeno-Kang (TSK) fuzzy system is formed by k fuzzy inference rules that have an "if-then" structure: the first part is called the antecedent and the second the consequent. In the consequent, we get the output of the rule, normally a linear function that depends on the input data and the learning parameters. The importance of the rule (weight) is calculated in the antecedents part, initially, the input data are fuzzified to later compute the weight.

In the literature, the operator product is used to compute the weights. The new proposed model is obtained by replacing the product operator by the Choquet integral and its generalizations. The fuzzy measures that are used with the Choquet integral are uniform measure, relative measure, product measure, and power measure [6].

The performance of ML-TSKC FS is evaluated using benchmark Multi-label datasets from MULAN. Preliminary results using datasets such as Birds, Flags, and CAL500 together with the Choquet integral showed that our methodology outperformed the methods in the literature.

Acknowledgements . This research is funded by FAPERGS/Brazil (Proc. 19/2551-0001660-3), CNPq/Brazil (301618/2019-4, 305805/2021-5), Agencia Estatal de Investigación (Government of Spain) under the project PID2019-108392GB-I00 (AEI/10.13039/501100011033), CNPq/FAPERGS (Edital 07/2022 - Program to Support the Settlement of Young Doctors in Brazil). Spanish Ministry Science and Tech. (grant numbers: TIN2016-77356-P, PID2019-108392GB I00 (AEI/10.13039/501100011033))

References

1. Bustince, H., Mesiar, R., Fernandez, J., Galar, M., Paternain, D., Altalhi, A., Dimuro, G., Bedregal, B., Takác, Z.: d-choquet integrals: Choquet integrals based on dissimilarities. Fuzzy Sets and Systems **414**, 1–27 (2021). https://doi.org/https://doi.org/10.1016/j.fss.2020.03.019
2. Choquet, G.: Theory of capacities. Annales de l'Institut Fourier **5**, 131–295 (1953–1954)
3. Dimuro, G.P., Fernandez, J., Bedregal, B., Mesiar, R., Sanz, J.A., Lucca, G., Bustince, H.: The state-of-art of the generalizations of the Choquet integral: From aggregation and pre-aggregation to ordered directionally monotone functions. Information Fusion **57**, 27 – 43 (2020). https://doi.org/https://doi.org/10.1016/j.inffus.2019.10.005
4. Ferrero-Jaurrieta, M., Takac, Z., Fernandez, J., Horanska, L., Dimuro, G.P., Montes, S., Diaz, I., Bustince, H.: VCI-LSTM: Vector choquet integral-based long short-term memory. IEEE Transactions on Fuzzy Systems pp. 1–14 (2022). https://doi.org/10.1109/TFUZZ.2022.3222035

5. Lou, Q., Deng, Z., Xiao, Z., Choi, K.S., Wang, S.: Multilabel takagi-sugeno-kang fuzzy system. IEEE Transactions on Fuzzy Systems **30**(9), 3410–3425 (2022). https://doi.org/10.1109/TFUZZ.2021.3115967
6. Lucca, G., Sanz, J., Pereira Dimuro, G., Bedregal, B., Mesiar, R., Kolesarova, A., Bustince Sola, H.: Pre-aggregation functions: construction and an application. IEEE Transactions on Fuzzy Systems **24**(2), 260–272 (April 2016). https://doi.org/10.1109/TFUZZ.2015.2453020
7. Wieczynski, J., Fumanal-Idocin, J., Lucca, G., Borges, E.N., Asmus, T.d.C., Emmendorfer, L.R., Bustince, H., Dimuro, G.P.: d-xc integrals: On the generalization of the expanded form of the choquet integral by restricted dissimilarity functions and their applications. IEEE Transactions on Fuzzy Systems **30**(12), 5376–5389 (2022). https://doi.org/10.1109/TFUZZ.2022.3176916
8. Wieczynski, J., Lucca, G., Dimuro, G.P., Borges, E.N., Sanz, J.A., Asmus, T.d.C., Fernandez, J., Bustince, H.: dc_F-integrals: Generalizing c_F-integrals by means of restricted dissimilarity functions. IEEE Transactions on Fuzzy Systems **31**(1), 160–173 (2023). https://doi.org/10.1109/TFUZZ.2022.3184054

Multivalued data fusion by means of a selection of maximal admissible permutations *

Mikel Ferrero-Jaurrieta[1], Zdenko Takáč[2], Ľubomira Horanská[2], Radko Mesiar[2], Mária Minárova[2], Javier Fernandez[1], and Humberto Bustince[1]

[1] Department of Statistics, Computer Science and Mathematics, Public University of Navarra, Campus Arrosadia s/n, 31006 Pamplona, Spain
{mikel.ferrero, fcojavier.fernandez, bustince}@unavarra.es
[2] Slovak University of Technology in Bratislava, Radlinského 9, 81005 Bratislava, Slovakia
{zdenko.takac, lubomira.horanska, maria.minarova}@stuba.sk,mesiar@math.sk

Keywords: Aggregation function · Multivalued data · Admissible permutations · Degree of totalness

Multivalued information fusion is a fundamental phase in any artificial intelligence process. Examples include the fusion of information from different sources, the aggregation of neural network features, the pre-processing of data for the selection of variables from a database, etc.

Some aggregation functions, such as the Choquet [1] or Sugeno integral, require sorting of the elements to be aggregated. However, considering that the information in question is multivalued, this intermediate task is not minor.

Admissible orders [2] were introduced for the purpose of ordering multivalued information as total orders refining the given partial order. However, the number of all admissible orders for a given ordinal structure can be very large and, in addition, the problem of selecting an admissible order which is suitable for a particular aggregation process prevails.

For a given subset of a partially ordered set so-called admissible permutations were introduced in [3] as restrictions of admissible orders to that subset. In this work we introduce the degree of totalness which can be assigned to each admissible permutation. The notion of degree of totalness fuzzifies the notion of partial order which is a crisp relation, and it measures a degree in which this relation holds.

We obtain a set of admissible permutations with maximal degree of totalness. Finally, the aggregation function is calculated for each of those admissible permutation and then the results are aggregated by means of a symmetric aggregation function.

References

1. Choquet, G. "Théorie des capacités." Annales de l'Institut Fourier. Vol. 5. 1953.
2. De Miguel, L., Sesma-Sara, M., Elkano, M., Asiain, M., Bustince, H. "An Algorithm for group decision making using n-dimensional fuzzy sets, admissible orders and OWA operators". Information Fusion 37 (2017) 126-131.
3. Paternain, D., De Miguel, L., Ochoa, G., Lizasoain, I., Mesiar, R., Bustince, H. "The Interval-Valued Choquet Integral Based on Admissible Permutations". IEEE Transactions on Fuzzy Systems, vol. 27, no. 8, pp. 1638-1647, Aug. 2019, doi: 10.1109/TFUZZ.2018.2886157.

* Supported by research project PID2019-108392GB-I00 (AEI/10.13039/ 501100011033) of the Agencia Estatal de Investigación and Grant VEGA 1/0267/21, VEGA 1/0036/23, APVV-18-0052, and by Tracasa Instrumental and the Immigration Policy and Justice Department of the Government of Navarre.

Fuzzy equivalences and aggregation functions in data exploration

Anna Król, Wojciech Rząsa, and Piotr Grochowalski

University of Rzeszów, Rejtana 16c, 35-959 Rzeszów, Poland {akrol,wrzasa,pgrochowalski}@ur.edu.pl

Keywords: Fuzzy equivalence · Aggregation function · k-NN algorithm.

In this contribution we deal with notions of fuzzy equivalence connectives, their aggregations, and their applicability to data exploration. We study the performance of families of fuzzy equivalences, such as Fodor-Roubens equivalences [1]. We use compositions of aggregations and fuzzy equivalences as a closeness measure in some data mining problems where distance-based algorithms were used so far. We consider such defined measure of closeness of objects of multidimensional spaces in classification by a k-nearest neighbour classifier and a modified kNN algorithm, called radius kNN algorithm with aggregations of fuzzy equivalences, where selection of the nearest neighbours is limited by their closeness from a tested object. This is a continuation of the research presented in [2,3] where usefulness of closeness measures by their comparison with metrics in kNN algorithm was proved. Some versions of radius nearest neighbour algorithm can be found e.g. in [4,5].

References

1. Fodor, J., Roubens, M.: Fuzzy preference modelling and multicriteria decision Support, Kluwer, Dordrecht (1994)
2. Król, A., Rząsa W., Grochowalski P.: Aggregation of Fuzzy Equivalences in Data Exploration by kNN Classifier, IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2020), Glasgow, UK, July 19-24, 2020, pp. 1-8 (2020)
3. Grochowalski P., Król, A., Rząsa W.: Radius kNN Classifier Using Aggregation of Fuzzy Equivalences, 2021 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), July 11-14, 2021, Luxembourg, Luxembourg, 1-6 (2021)
4. Zhu, Y., Wang, Z., Gao, D.: Gravitational fixed radius nearest neighbor for imbalanced problem, Knowledge-Based Systems 90, 224-238 (2015)
5. Wang, Z., Li, Y., Li, D., Zhu Z., Du, W.: Entropy and gravitation based dynamic radius nearest neighbor classification for imbalanced problem, Knowledge-Based Systems 193, Article 105474, 1-14 (2020)

Choosing admisible permutations

Xabier González-García[1], Mikel Ferrero-Jaurrieta[1], Ľubomïra Horanská[2], Zdenko Takáč[2], and Humberto Bustince[1]

[1] Department of Statistics, Computer Science and Mathematics, Public University of Navarre, Campus Arrosadía, s/n, 31006 Pamplona, Spain.
[2] Slovak University of Technology in Bratislava, Institute of Information Engineering, Automation and Mathematics, Radlinskeho 9, 81237, Bratislava, Slovakia
{xabier.gonzalez.garcia,mikel.ferrero,bustince}@unavarra.es,
{lubomira.horanska,zdenko.takac}@@stuba.sk

Keywords: Information fusion, Data ordering, Admissible permutations

Ordering data is a crucial step in the process of information fusion, where multiple sources of information are merged to extract accurate and complete information. Two common ways of establishing a consistent sequence of events or values are total and partial ordering. A total order is a binary relation on a set that satisfies the properties of reflexivity, antisymmetry, and transitivity, meaning that every pair of elements can be compared and arranged in a linear sequence, whereas in partial orders the last property is not required.

Admissible orders are a type of total order that refines a partial order by providing a more general, branching structure that can capture complex relationships among data sources [1]. They have several advantages over both partial and total orders.

Firstly, admissible orders are more flexible than total orders, which can be overly restrictive in situations where the relationships among data sources are complex and cannot be easily reduced to a linear ordering. Secondly, admissible orders provide more consistency than partial orders by ensuring that the resulting merged data is free from conflicts or redundancies.

There are infinitely many admissible orders, and in fact, many of these orders are equivalent, resulting in the same arrangement for a given vector of elements [3]. Moreover an admissible permutation is needed to order a dataset according to the existing order. However, the problem of finding all admissible permutations of a dataset can be challenging, as the number of permutations of n elements is $n!$. Therefore, developing new methods for efficiently choosing admissible permutations is important for information fusion applications [2]. These methods can improve the speed and accuracy of the data fusion process, enabling more efficient and effective decision-making. In this work we propose a novel method for choosing admissible permutations.

Acknowledgement

This work has been funded by the Agencia Estatal de Investigación (Government of Spain) under the project PID2019-108392GB-I00 (AEI/10.13039/501100011033), Tracasa Instrumental and the grant VEGA 1/0267/21.

References

1. H Bustince, Javier Fernández, Anna Kolesárová, and Radko Mesiar. Generation of linear orders for intervals by means of aggregation functions. *Fuzzy Sets and Systems*, 220:69–77, 2013.
2. Mikel Ferrero-Jaurrieta, L'ubomíra Horanská, Julio Lafuente, Radko Mesiar, Graçaliz Pereira Dimuro, Zdenko Takáč, Marisol Gómez, Javier Fernández, and Humberto Bustince. Degree of totalness: How to choose the best admissible permutation for vector fuzzy integration. *Fuzzy Sets and Systems*, 2023.
3. Daniel Paternain, Laura De Miguel, Gustavo Ochoa, Inmaculada Lizasoain, Radko Mesiar, and Humberto Bustince. The interval-valued choquet integral based on admissible permutations. *IEEE Transactions on Fuzzy Systems*, 27(8):1638–1647, 2018.

Part VI

SS3: Evaluative linguistic expressions, generalized quantifiers and applications

Complexity and Universality of Evaluative Expressions

Antoni Brosa-Rodríguez, Susana M. Campillo-Muñoz, M. Dolores Jiménez-López, and Adrià Torrens-Urrutia

GRLMC-Research Group in Mathematical Linguistics
Universitat Rovira i Virgili,
Tarragona, 43002, Spain
{antoni.brosa, susanamaria.campillo, mariadolores.jimenez, adria.torrens}@urv.cat

We present a study of the complexity of evaluative expressions through the analysis of these structures in a representative set of natural languages. Linguistic complexity is a vague concept that should be studied within a fuzzy model. The complexity of natural language expressions can be measured in terms of their degree of universality: the more universal an expression is, the less complex it is.

Our proposal has a balanced set of evaluative expressions from different languages (Arabic, Catalan, Hungarian, Thai, Vietnamese, West Greenlandic and Icelandic). This set represents different families, genera, macro-areas and linguistic typologies. Each language in the set has different properties in its evaluative expressions. Taking into account the presence of each property in the whole set, we assign different degrees of universality to the properties. From the degree of universality, we can measure the level of complexity of evaluative expressions in each language. If the evaluative expression in a given language presents properties with a low degree of universality, then its complexity will be high. Conversely, if the properties of the evaluative expressions in a given language have a high degree of universality, then their complexity will be low.

For each language in the set, have studied the following variables of evaluative expressions: the presence (or not) of all the possible elements of an evaluative expression; the agreement (or not) of the elements of the expression; the order of the elements; the grammatical category of those elements and other language-specific characteristics. The sum of the analysis of all these variables allows us to establish the fuzzy relationship between universality and complexity mentioned above.

Our results show three key ideas: Firstly, we can establish three different clusters of complexity: languages with high levels of complexity, languages with a medium level of complexity and languages with a low level of complexity. Secondly, our definition of an evaluative expression is validated: the compulsory elements in any evaluative expression are the referent and the head. Finally, some languages have a nexus between the referent and the head of the evaluation as a mandatory element. In contrast, the exclusion of this nexus is a must in another group of languages.

The next step will be to implement this model to generate small and specific fuzzy universal grammar systems for the processing of only evaluative expressions. This type of fuzzy analysis may have many interesting applications in natural language processing, such as self-learning language tools, opinion mining, author profiling, etc.

Acknowledgement

Grant PID2020-120158GB-I00 funded by MCIN/AEI/ 10.13039/501100011033.

References

1. Novák, V.: A comprehensive theory of trichotomous evaluative linguistic expressions. Fuzzy sets and systems, **159**(22), 2939-2969 (2008).
2. Novak, V.: Evaluative linguistic expressions vs. fuzzy categories. Fuzzy Sets and Systems, **281**, 73-8 (2015).
3. Torrens-Urrutia, A., Jiménez-López, M. D., Brosa-Rodríguez, A., & Adamczyk, D.: A Fuzzy Grammar for Evaluating Universality and Complexity in Natural Language. Mathematics, **10**(15), 2602 (2022)
4. Torrens-Urrutia, A., Novák, V., & Jiménez-López, M. D.: Describing Linguistic Vagueness of Evaluative Expressions Using Fuzzy Natural Logic and Linguistic Constraints. Mathematics, **10**(15), 2760 (2022).
5. Dryer, M. S. & Haspelmath, M. (2013). The World Atlas of Language Structures Online. http://wals.info. Last accessed 01/04/2023

Part VII

SS4: Neural networks under uncertainty and imperfect information

Managing uncertainty in Deep Learning architectures through Interval-valued features

Iosu Rodriguez-Martinez[1], Xabier Gonzalez-Garcia[1], Jonata Wieczynski[1], Francisco Herrera[2], Zdenko Takáč[3], and Humberto Bustince[1]

[1] Departamento de Estadística, Informática y Matemáticas, Universidad Pública de Navarra, Campus Arrosadia s/n, 31006, Pamplona, Spain
[2] Instituto Andaluz Interuniversitario en Data Science and Computational Intelligence, Calle Periodista Daniel Saucedo Aranda, s/n, 18071, Granada, Spain
[3] Slovak University of Technology in Bratislava, Institute of Information Engineering, Automation and Mathematics, Radlinskeho 9, 81237, Bratislava, Slovakia

Uncertainty is fairly common in deep-learning applications. On the one hand, measurement errors can include noise in the input data which may propagate through all steps of the learning process. On the other, the black-box nature of these models confers a distrustful nature to their predictions. However, in practice, most state-of-the-art models ignore these problems, which may limit their application in environments where guarantees are expected.

In this work, we present a Convolutional Neural Network in which the intermediate features handled by the model are represented by intervals. Interval-valued data representations have been shown to be useful in We find that this idea confers two important benefits. Firstly, anomalies or "hard" samples can be detected based on the width of the generated intervals. Secondly, during training, a regularization effect similar to techniques such as Dropout is achieved if we operate with random values sampled from within the intervals.

Similar ideas have already been proposed in the past. Interval predictions generated by different machine learning models have been used for tasks such as modeling the uncertainty provided by different classifiers in an ensemble process [3]. In the specific case of neural networks, some authors have tried modeling their inputs and parameters of through Interval Valued Fuzzy Sets instead of point values [2], as well as through intervals of real values [4]. Sadly, both of these approaches impose serious restrictions to the structure and learning process of the models in order to preserve the mathematical correctness of the intervals. In contrast, our approach can be directly incorporated in modern deep-learning models and optimization pipelines. More similar to our approach, conformal prediction [1] is a technique based on outputting confidence-intervals in order to guarantee the reliability of a model's predictions. However, it requires an additional subset of samples for "callibrating" the model, and does not consider the uncertainty associated to the features of the model during training.

Acknowledgements

This work was supported by the project PID2019-108392GB-I00 (AEI/10.13039/501100011033) of the Spanish Ministry of Science, Tracasa Instrumental, the Andalusian Excellence project P18-FR4961 and the grant VEGA 1/0267/21 and the Navarra Artificial Intelligence Research Center.

References

1. Anastasios N Angelopoulos and Stephen Bates. A gentle introduction to conformal prediction and distribution-free uncertainty quantification. *arXiv preprint arXiv:2107.07511*, 2021.
2. James J Buckley and Yoichi Hayashi. Fuzzy neural networks: A survey. *Fuzzy sets and systems*, 66(1):1–13, 1994.
3. J Fumanal-Idocin, Z Takáč, L' Horanská, T da Cruz Asmus, G Dimuro, C Vidaurre, J Fernandez, and H Bustince. A generalization of the sugeno integral to aggregate interval-valued data: An application to brain computer interface and social network analysis. *Fuzzy Sets and Systems*, 451:320–341, 2022.
4. Antonio Muñoz San Roque, Carlos Maté, Javier Arroyo, and Ángel Sarabia. imlp: Applying multi-layer perceptrons to interval-valued data. *Neural Processing Letters*, 25(2):157–169, 2007.

Flood detection due to river overflowing using AI

Iñaki Pérez del Notario López[1,3], Humberto Bustince Sola[1], and Peio Oria Iriarte[2]

[1] Public university of Navarre (UPNA), Pamplona, Spain
[2] Meteorology statal agency (AEMET), Spain
[3] Tesicnor S.L., Noain, Spain

Keywords: Neural Networks · Time series prediction · Takagi-Sugeno · Risk prevention.

The floodings are the most common type of natural disaster in the word, this kind of disaster causes a huge amount of damage on properties like buildings, streets, cars, farmlands, etc. And the unfortunate loss of human lives. All of this, leading to great economic costs and mental health problems on the citizens.

Dealing with this type of disasters is a challenge that we have to face as a society, but to do it, people have to be aware of their possibility of occurrence and that there are ways to greatly prevent the damage applying appropriate measures, like deploying anti-flood barriers, relocating vehicles, evacuating people, etc.

In order to do this, we are using artificial intelligence to predict when a flood is likely to occur due to an overflow of the water level on a river. In this case, we are studying the Arga river that passes through Pamplona because historically has caused episodes of water invading residential zones, causing a lot of damage. To feed the models, we are using data of the state of the river with variables like the water level, supplied by hydrographic confederation of the Ebro river (CHEBRO) and atmospheric variables that can impact on the water level in some way, like the precipitation, the soil moisture, temperature, and future predictions of precipitations, all of this supplied by the meteorology statal agency (AEMET).

To make predictions with temporal series, we are using two types of models. On one hand, we are using recurrent neural networks because this type of models takes into consideration the evolution of the data through time, and that can be helpful to detect patterns that can lead to a sudden increase on the water level that would be more difficult to detect with traditional regression models. On the other hand, we also are using models based on Takagi-Sugeno inference system (TSK) because, with these types of models, fuzzy rules can be generated that allows us a better understanding of how the model is doing a certain prediction, and the results can be verified with experts on hydrology. Taking into account that the final decision of raising an alarm has to be made by humans, knowing how the model works can be very helpful.

The fast flood predictions is a difficult problem due to the low frequency of occurrence and the chaotic behaviour of the meteorology and, with the effects of the climate change, this incidents are going to be more common and severe. Due to this fact, it's important for us to be able to detect them and be prepared when they come to make a safer future.

Fuzzy Partitions in Terms of Feature Maps of Reproducing Kernel Hilbert Spaces

Irina Perfilieva

Institute for Research and Applications of Fuzzy Modeling,
University of Ostrava, 30.
dubna 22, 701 03 Ostrava, Czech Republic,
Irina.Perfilieva@osu.cz

In this contribution, we show that one of the possible approaches to the definition of a fuzzy partition (especially in multidimensional space) is to relate this concept to a positive definite (p-d) kernel [1] and thereby define the fuzzy partition elements as its (p-d kernel) feature maps. On this theoretical basis, we can use the close relationship between p-d kernels and reproducing kernel Hilbert spaces (RKHS) and their feature maps.

A constructive approach from a fuzzy partition through the kernel associated with it to the unique Hilbert space, for which this kernel is reproducing, is based on the Moore-Aronszajn theorem [2]. It can be shown in more detail that the definition of a fuzzy partition can be uniquely related to the p-d-symmetric kernel and thus defines a unique RKHS whose only reproducing kernel is it.

The importance of the proposed theoretical construction lies in its connection with the so-called Representer Theorems [3], which help to reduce the general problem of machine learning to algorithms implemented on computers. To give some details, we note that, based on the Representer Theorem, each minimizing function in an RKHS can be written as a linear combination of a kernel function evaluated at training points. This fact significantly simplifies the problem of minimizing the empirical risk from an infinite-dimensional to a finite-dimensional optimization problem.

References

1. Patane, G.: Continuous Fuzzy Transform as Integral Operator, IEEE TFS, 29, 3093 – 3102 (2021).
2. Aronszajn, N.: Theory of Reproducing Kernels, Trans. American Math. Soc., 68, 337 – 404 (1950).
3. Scholkopf, B., Herbrich, R., Smola, A.: A Generalized Representer Theorem. Computational Learning Theory, LNCS 2111, 416 – 426 (2001).

Feature uncertainty management using intervals in Recurrent Neural Networks*

A. Indurain[1], M. Ferrero-Jaurrieta[1], Z. Takác[2], I. Rodriguez-Martinez[1], J. Fernandez[1], and H. Bustince[1]

[1] Public university of Navarra and Institute of Smart Cities, Campus Arrosadia s/n, 31006 Pamplona, Spain
{alfonso.indurain mikel.ferrero iosu.rodriguez fcojavier.fernandez bustince}@unavarra.es
[2] Slovak University of Technology in Bratislava zdenko.takac@stuba.sk

Keywords: Recurrent neural networks · Feature uncertainty · Interval uncertainty · Sentiment analysis.

Recurrent neural networks are a type of neural networks that have the ability to model temporal information. Examples are Long Short-Term Memory (LSTM) [2]. A fundamental task of these networks is therefore the extraction of temporal features for pattern recognition.

It is often common to use information with associated uncertainty. For example, data that have already been preprocessed, use of features extracted by a model, intrinsic uncertainty associated with the type of data, etc.

In this work we consider preprocessed audio, video and text datasets [3] with associated uncertainty. To model this uncertainty we use intervals, which are created from the distribution of the data. Once we deal with uncertainty using intervals, we claim that we can obtain better results than those without considering uncertainty. To contract the intervals and obtain a midpoint within the intervals, we use k-alpha [1] operators, where the parameter k is learned by the recurrent neural network through the Adam optimiser.

References

1. Bustince, H., Calvo, T., De Baets, B., Fodor, J., Mesiar, M., Montero, J., Paternain, D., Pradera, A. "A class of aggregation functions encompassing two-dimensional OWA operators", Information Sciences, Vol. 180, 10, (2010) 1977-1989.
2. Hochreiter, S., Schmidhuber, J; Long Short-Term Memory. Neural Computation 1997; 9 (8): 1735–1780
3. Wang, Y., Shen, Y., Liu, Z., Liang, P. P., Zadeh, A., Morency, L.-P. (2019). Words Can Shift: Dynamically Adjusting Word Representations Using Nonverbal Behaviors. Proceedings of the AAAI Conference on Artificial Intelligence, 33(01), 7216-7223.

* This work was supported by research project PID2019-108392GB-I00 (AEI/10.13039/501100011033) of the Agencia Estatal de Investigaci´on and Grant VEGA 1/0545/20., and by Tracasa Instrumental and the Inmigration Polizy and Justice Department of the Government of Navarre

Part VIII

SS5: Imprecision modeling and management in XAI systems

Optimizing performance and resiliency against small perturbations in classification problems

Javier Fumanal-Idocin[1], Humberto Bustince[1], Javier Andreu-Perez[2,3], and Hani Hagras[2]

[1] Departamento de Estadística, Informática y Matemáticas, Universidad Pública de Navarra, Campus Arrosadia s/n, 31006, Pamplona, Spain
[2] Centre for Computational Intelligence, School of Computer Science and Electronic Engineering, 18071, Essex, United Kingdom
[3] Simbad2, University of Jaen, Jaen, Spain

Tabular data classification is one of the most popular tasks performed using machine learning [2,4], and is also a very researched topic in the fuzzy community [3]. Besides the very-well known benchmark datasets, classifiers need to be reliable in real-world environments to be useful. This research focuses on the study of data classification and the behaviour of different crisp and fuzzy classifiers in response to small changes in input data. The aim is to identify the most suitable classifiers that can handle small variations in data while still maintaining accurate results. We also discuss the concept of "small" change and how it changes depending on the data domain. These considerations are used to discuss some problems in explainable classification, as the reliability of predictions in border cases. Subsequently, we propose a new metric to measure this, using the changes in predictions in a validation set. Then, we propose an optimization method to improve its performance using a genetic algorithm. Our findings suggest that fuzzy classifiers might exhibit a smoother performance in this sense depending on the type of fuzzy sets used. As interval estimations of fuzzy memberships can be more more natural to express for humans, we aim for systems where interval fuzzy memberships can lead to both more accurate and reliable predictions, specially in border cases between classes [1].

Acknowledgments

This work was supported in part by Oracle Cloud credits and related re- sources provided by Oracle for Research.

References

1. Javier Andreu-Perez, Lauren L Emberson, Mehrin Kiani, Maria Laura Filippetti, Hani Hagras, and Silvia Rigato. Explainable artificial intelligence based analysis for interpreting infant fnirs data in developmental cognitive neuroscience. *Communications biology*, 4(1):1077, 2021.
2. J Fumanal-Idocin, O Cordón, and H Bustince. The krypteia ensemble: Designing classifier ensembles using an ancient spartan military tradition. *Information Fusion*, 90:283–297, 2023.
3. Mehrin Kiani, Javier Andreu-Perez, and Hani Hagras. A temporal type-2 fuzzy system for time-dependent explainable artificial intelligence. *IEEE Transactions on Artificial Intelligence*, pages 1–15, 2022.
4. Sotiris B Kotsiantis, Ioannis Zaharakis, P Pintelas, et al. Supervised machine learning: A review of classification techniques. *Emerging artificial intelligence applications in computer engineering*, 160(1):3–24, 2007.

Training hierarchical fuzzy systems to predict shipbreaking and shipbeaching on real world ILT data

Lynn Pickering[1], Victor Ciulei[2], Paul Merkx[2], Bernard De Baets[3], and Kelly Cohen[1]

[1] Department of Aerospace Engineering and Engineering Mechanics, University of Cincinnati, Cincinnati OH, USA
[2] The Human Environment and Transport Inspectorate (ILT), Netherlands
[3] Faculty of Bioscience Engineering, Gent University, Gent, Belgium

Keywords: genetic fuzzy system · fuzzy system · interpretable machine learning · hierarchical fuzzy system

The goal of this work is to design a fuzzy logic system as an interpretable decision support system. This is a first step towards an instrument that may be used by waste inspectors (end users) for the prevention of illegal shipbreaking. We train a Hierarchical Fuzzy System (HFS) using a Genetic Algorithm (GA) on two connected applications; a Shipbreaking application and a Shipbeaching application. The data is collected by the Dutch Human Environment and Transport Inspectorate (ILT) from real ships. We examine several iterations of design choices which improve the HFS system. The final three layer hierarchy is chosen to maximize interpretability of the model by reducing the number of rules needed to describe the system, as well as mimicking a structure that humans might group the set of inputs into [2]. The trained HFS is compared to single tree and random forest machine learning models previously trained on this data. In our comparison we look at the area under the precision-recall curve, and attempt to analyze the difference in interpretability of the model outputs. The area under the precision-recall curve is comparable for the trained HFS and a single tree model limited to the same depth as the HFS, while the random forest model outperforms both by a sizeable amount. The HFS, single tree and random forest models are explained using available methods for interpreting black box models (primarily SHAP (SHapley Additive exPlanations) [1]). Further, a decision visualization process is introduced to provide a higher interpretability to the HFS.

Acknowledgements

Lynn Pickering acknowledges the support of the Rindsberg Fellowship from the University of Cincinnati, the Ohio Space Grant Consortium Research Fellowship, and a Fulbright/Ghent University Award.

References

1. Scott M Lundberg and Su-In Lee. A unified approach to interpreting model predictions. *Advances in neural information processing systems*, 30, 2017.
2. Luis Magdalena. Designing interpretable hierarchical fuzzy systems. In *2018 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, pages 1–8. IEEE, 2018.

Part IX

SS6: Recent trends in mathematical fuzzy logics

Join irreducible varieties of residuated lattices

Paolo Aglianò[1] and Sara Ugolini[2]

[1] Università di Siena, Siena, Italy agliano@live.com

[2] Artificial Intelligence Research Institute (IIIA), CSIC, Barcelona, Spain sara@iiia.csic.es

Substructural logics constitute a large class of logical systems algebraizable in the sense of Blok-Pigozzi: they encompass classical logic, intermediate logics, fuzzy logics, relevance logics and many other systems, all seen as logical extensions of the Full Lambek calculus $\mathcal{FL}$. Here we are interested in the positive fragment of $\mathcal{FL}$ (the system obtained by removing the constant 0, and consequently negation, from the language), $\mathcal{FL}^+$, whose corresponding algebraic semantics is given by the variety of residuated lattices RL. We will characterize join irreducibility in a large class of residuated lattices; to do so we will adapt to our purpose part of the theory developed in [2] about satisfaction of formulas generated by iterated conjugates.

We define a set $B^n(x, y)$ of equations in two variables x, y for all $n \in \mathbb{N}$ in the following way; let Γ^n be the set of iterated conjugates of *length* n (i.e. a composition of n left and right conjugates) over the appropriate language, with $\Gamma^0 = \{l_1\}$ (for a more general definition, here not needed, see [2], page 229). For all $n \in \mathbb{N}$

$$B^n(x, y) = \{\gamma_1(x) \vee \gamma_2(y) \approx 1 \,:\, \gamma_1, \gamma_2 \in \Gamma^n\}.$$

Let $\mathbf{A}$ be a residuated lattice and $a, b \in A$; we say that $\mathbf{A}$ **satisfies** $B^n(a, b)$, in symbols $\mathbf{A} \vDash B^n(a, b)$ if $\mathbf{A}, a, b \vDash B^n(x, y)$. i.e. $\gamma_1(a) \vee \gamma_2(b) = 1$ for all $\gamma_1, \gamma_2 \in \Gamma^n(\mathbf{A})$. We say that $\mathbf{A}$ satisfies $(G_{n,k})$ if for all $a, b \in A$, if $\mathbf{A} \vDash B^n(a, b)$, then $\mathbf{A} \vDash B^k(a, b)$.

Theorem 1. *[1] Let V be a variety of residuated lattices that satisfies $(G_{n,n+1})$ for some $n \in \mathbb{N}$; V is join irreducible if and only if there is a subdirectly irreducible algebra $\mathbf{B} \in \mathsf{V}$ such that $\mathbf{V}(\mathbf{B}) = \mathsf{V}$.*

Using the fact that all normal varieties and all representable varieties satisfy the hypotheses of Theorem 1, one can (among other things) characterize all strictly join irreducible varieties of basic hoops and all linear varieties of basic hoops. Finally we point out that all the material covered in this abstract appeared in [1].

References

1. P. Aglianò and S. Ugolini, *Strictly join irreducible varieties of residuated lattices*, J. Logic Comput. **32** (2022), 32–64.
2. N. Galatos, *Equational Bases for Joins of Residuated-lattice Varieties*, Studia Logica **76** (2004), 227–240.

A software for dealing with Gödel and Nilpontent Minimum logic

Stefano Aguzzoli[1], Brunella Gerla[2], and Paolo Pantaleo[2]

[1] Dipartimento di Informatica, Università di Milano, Italy stefano.aguzzoli@unimi.it
[2] Dipartimento di Scienze Teoriche e Applicate, Università dell'Insubria, Italy
brunella.gerla@uninsubria.it

In classical propositional logic, truth tables of formulas with n variables are exactly all the Boolean functions from the n-th power of {0,1} into {0,1} and hence coincide with subsets of the n-th power of {0,1}, that is they form the free Boolean algebra over n generators.

When dealing with many-valued logics, the set D of truth values is generally bigger than {0,1} and in many cases there are no results of functional completeness, meaning that not all the formulas from the n-th power of D to D are truth tables of formulas.

We consider here the case of two logical systems, Godel-Dummet logic and Nilpotent Minimum logic, which have the following properties: even if we consider as set of truth values the infinite set [0,1], the set of all possible truth tables of formulas with a finite number of variables is finite; just as free Boolean algebras are powersets of some set, the free algebraic structures corresponding to Godel-Dummet logic and Nilpotent Minimum logic can be constructed collecting downsets of forests (instead of subsets of sets).

In this context, boolean functions can be hence seen as subsets of a set made by truth assignments of variables; we expand such an approach by considering downsets of a forest as a way to add more nuances of truth values for each assignment of variables (we can consider the root of each tree in the forest as a crisp truth assignment, and the descendant of such root as nuances of that truth assignment). The constraints and the interpretation of such behaviour depend on the particular kind of logic that we consider.

Starting from [1] and [2], we present here a software that collects tools for dealing with combinatorial aspects related to Godel logic and NM-logic. We developed tools to build algebras starting from duals, and to graphically represent both of them. Further, we calculate duals of free Godel algebras and NM-algebras, by implementing the operation of product between forests in the appropriate category. Finally, we show how to visualize the interpretation of any formula as the proper subforest of the free algebra. The software, written in Java, can be downloaded from [3].

References

1. Aguzzoli S., Bova S., Gerla B. (2011) "Free algebras and functional representation for fuzzy logics". In: Cintula P, Hájek P, Noguera C (eds) Handbook of mathematical fuzzy logic, vol 2. College Publications, Cambridge, pp 713-791.
2. Aguzzoli S., Busaniche M. and Marra V., (2007) "Spectral Duality for Finitely Generated Nilpotent Minimum Algebras, with Applications", in Journal of Logic and Computation, vol. 17, no. 4, pp. 749-765.
3. https://github.com/panta1005/ForestOperator

Fuzzy-tolerance based rough set approach for Feature Selection in Set-valued information system

Shivani Singh[1] and Niladri Chatterjee[2]

[1] Indian Institute of Technology Delhi
[2] School of Information Technology, IIT Delhi

Keywords: Set-valued data, Rough set, Fuzzy tolerance relation, Feature selection

Databases obtained from different search engines, market data, patients' symptoms and behaviours, etc., are some common examples of set-valued data, in which a set of values are correlated with a single entity [1][2]. In real-world data deluge, numerous irrelevant attributes lower the ability of experts both in speed and in predictive accuracy due to high dimension and insignificant information, respectively. Feature selection is the concept of selecting those attributes that ideally are necessary as well as sufficient to better describe the target knowledge. Rough set-based methods can handle uncertainty presented in the real-valued information systems but after discretization process when applied to real-data [3]. Dubois and Prade [4] combines fuzzy set [5] with rough set [3] and proposed a fuzzy rough set to provide an important tool in reasoning with uncertainty for real-valued datasets. In our research work, we introduce a novel approach for feature selection in set-valued information system based on tolerance rough set theory. The fuzzy tolerance relation between two objects using a similarity threshold is defined. We find reducts based on the degree of dependency method for selecting best subsets of attributes in order to obtain higher knowledge from the information system. Feature selection in an incomplete information system is also undertaken by means of transforming the incomplete information system into set-valued information system.

References

1. Qian, Y., Dang, C., Liang, J., Tang, D.: Set-valued ordered information systems. Inf Sci 179(16), 2809–2832 (2009).
2. Dai, J., Tian, H.: Fuzzy rough set model for set-valued data. Fuzzy Sets Syst 229, 54–68 (2013).
3. Pawlak, Z., Skowron, A.: Rudiments of rough sets. Inf Sci 177(1), 3–27 (2007).
4. Dubois, D., Prade, H.: Putting rough sets and fuzzy sets together. In: Słowinski R (ed) Intelligent decision support. Springer, Dordrecht, pp 203–232 (1992).
5. Zadeh, LA.: Fuzzy sets. In Fuzzy sets, fuzzy logic, and fuzzy systems, selected papers by Lotfi A Zadeh, pp. 394–432 (1996).

Part X

SS7: Fuzzy graph-based models: theory and application

Fuzzy rough approximation operators and fuzzy relation equations

Jelena Ignjatović, Ivan Stanković, and Miroslav Ćirić

University of Niš, Faculty of Sciences and Mathematics, Višegradska 33, 18000 Niš, Serbia
{jelena.ignjatovic, ivan.stankovic, miroslav.ciric}@pmf.edu.rs

Rough set theory is an important mathematical tool for managing uncertainty and granularity in information systems introduced by Pawlak. It was a subject of extensive research in recent years (cf.,e.g., [2], [5], [18]). Rough sets are a type of mathematical structure that can be used to simplify complex data by eliminating irrelevant information, allowing for more efficient processing and analysis, and consequently, they are useful when dealing with incomplete or imperfect data, such as missing values or noisy data. The development of rough set theory and its various extensions has led to significant advancements in many areas of computer science. One of the strengths of rough set theory is its versatility, and because of that it was considered to be an efficient tool in various applications such as approximate classification, machine learning, conflict analysis, pattern recognition, data mining, and automated knowledge acquisition. In recent years, there has been a growing interest in the intersection of rough set theory and fuzzy set theory (cf.,e.g., [1], [7], [10], [13], [14], [15], [16]). Many of the approaches used in the rough set theory can be applied to fuzzy sets, many algorithms used in the rough set theory can be adapted to deal with fuzzy sets, and the concepts of lower and upper approximations can be extended to fuzzy sets as well (see [6], [12], [19]). These two fields are complementary, and so fuzzy rough set theory allows more nuanced analysis of data. Motivated by the studies of fuzziness and roughness in algebraic systems and lattices (cf.,e.g.,[3], [4], [8], [11], [17]), systems of fuzzy relation inequalities and equations with a given family of fuzzy relations and a variable taking its values in the collection of all fuzzy subsets of a given set A, as well as systems with a given family of fuzzy relations and two unknown fuzzy subsets of given sets A and B, were analysed in this paper. The systems of these inequalities and equations were studied, and conditions for their solvability, properties, structure of the set to their solutions were investigated. The differences between solutions of the systems of inequalities and equations were highlighted. Some basic properties of the lower and upper fuzzy rough approximations have been presented by providing algorithms for computing extremal solutions to the given systems. The main tools used in the paper are fuzzy quasi-orders (as well as fuzzy equivalences), residuals of fuzzy relations, and closures and openings on a lattice of fuzzy sets. The importance and applicability of provided algorithms are illustrated by their use in fuzzy automata theory, arising from the results provided in [9].

Funding statement:

This research was supported by the Science Fund of the Republic of Serbia, Grant no 7750185, Quantitative Automata Models: Fundamental Problems and Applications - QUAM

References

1. Bao, Y.L., Yang, H.L., She, Y.H.: Using one axiom to characterize L-fuzzy rough approximation operators based on residuated lattices. Fuzzy Sets Syst. **336**, 87–115 (2018)
2. Greco, S., Matarazzo, B., Slowinski, R.: Rough approximation by dominance relations. Int. J. Intell. Syst. **17**, 153–171 (2002)
3. Han, S.E., Kim, I.S., Šostak, A.: On approximate-type systems generated by L-relations. Inf. Sci. **281**, 8–21 (2014)
4. J. Ignjatović, M. Ćirić, B. Šešelja, A. Tapavčević: Fuzzy relation inequalities and equations, fuzzy quasi-orders, and closures and openings of fuzzy sets. Fuzzy sets and systems **260**, 1-24 (2015)
5. Kondo, M.: On the structure of generalized rough sets. Inf. Sci. **176**, 589–600 (2005)
6. Li, T.J., Leung, Y., Zhang, W.X.: Generalized Fuzzy Rough Approximation Operators Based on Fuzzy Coverings. Int. J. Approx. Reason. **48**, 836–856, (2009)
7. Liu, G.L.: Using one axiom to characterize rough set and fuzzy rough set approximations. Int. J. Approx. Reason.**80**, 348–358. (2017)
8. Micić, I., Jančić, Z., Stanimirović, S.: Computation of the greatest right and left invariant fuzzy quasi-orders and fuzzy equivalences. Fuzzy Sets and Systems **339**, 99–118 (2018)
9. Micić, I., Nguyen, L.A., Stanimirović, S.: Characterization and computation of approximate bisimulations for fuzzy automata. Fuzzy Sets and Systems **442**, 331–350 (2022)

10. Mi, J.S., Leung, Y., Zhao, H.Y., Feng, T.: Generalized fuzzy rough sets determined by a triangular norm. Inf. Sci. **178**, 3203–3213, (2008)
11. Mokoř, J.: Functors among Relational Variants of Categories Related to L-Fuzzy Partitions, L-Fuzzy Pretopological Spaces and L-Fuzzy Closure Spaces. Axioms **9**(63) (2020)
12. Pang, B., Mi, J.S., Yao, W.: L-fuzzy rough approximation operators via three new types of L-fuzzy relations. Soft Comput. **23**, 11433–11446 (2019)
13. Qiao, J.S., Hu, B.Q.: Granular variable recision L-fuzzy rough sets based on residuated lattices. Fuzzy Sets Syst. **336**, 148–166, (2018)
14. Qiao, J.S.; Hu, B.Q.: On $(\circ, \star)$-fuzzy rough sets based on residuated and co-residuated lattices. Fuzzy Sets Syst. 336, 54–86 (2018)
15. She, Y.H., Wang, G.J.: An axiomatic approach of fuzzy rough sets based on residuated lattices. Comput. Math. Appl. **58**, 189–201 (2009)
16. Wang, C.Y.; Zhang, X.G.; Wu, Y.H.: New results on single axiom for L-fuzzy rough approximation operators. Fuzzy Sets Syst. **380**, 131–149 (2020)
17. Wu, W.Z., Li, T.J., Gu, S.M.: Using one axiom to characterize fuzzy rough approximation operators determined by a fuzzy implication operator. Fundam. Inform. **142**, 87–104 (2015)
18. Zhu, W.: Relationship between generalized rough sets based on binary relation and covering. Inf. Sci. **179**, 210–225 (2009)
19. Zhao, F.F., Shi, F.G.: L-fuzzy generalized neighborhood system operator-based L-fuzzy approximation operators. Int. J. Gen. Syst. **50**, 458–484 (2021)

Part XI

SS9: Fuzzy implication functions

A new approach to subgroup discovery based on fuzzy implication functions

Raquel Fernandez-Peralta[1,2], Sebastia Massanet[1,2], and Balasubramaniam Jayaram[3]

[1] Soft Computing, Image Processing and Aggregation (SCOPIA) Research Group, Dept. Mathematics and Computer Science, University of the Balearic Islands, 07122 Palma, Spain
{r.fernandez,s.massanet}@uib.es
[2] Health Research Institute of the Balearic Islands (IdISBa), 07010 Palma, Spain
[3] Department of Mathematics, Indian Institute of Technology Hyderabad, Hyderabad - 502 285, India
jbala@iith.ac.in

Subgroup discovery (SD) is a widely known descriptive data mining technique designed for identifying subgroups of data which are interesting with respect to a target variable [1]. The importance of a subgroup is numerically quantified by a quality measure, which is selected according to the objectives of the task at hand. Each subgroup is normally represented in the form of a rule Condition $\rightarrow$ Target, where "Target" is the property of interest and "Condition" is a conjunction of features.

One of the key aspects in SD is the interpretability of the results, so the output should be simple enough to be understood and analysed by an expert. This requirement makes natural to consider the use of linguistic fuzzy rules to model subgroups. In accordance, several SD algorithms based on fuzzy logic have already been proposed in the literature [2]. However, up to our knowledge, these algorithms are only valid for categorical target variables and the rule form in the definition of a subgroup is interpreted as co-occurrence rather than a logical conditional. In this way, we propose a new approach that solves these two disadvantages by introducing the use of fuzzy implication functions.

Our contribution is to design an SD algorithm, specially designed for numeric target variables, based on linguistic fuzzy rules modelled by fuzzy implication functions. Due to the structure of these operators, the corresponding subgroups can be interpreted as conditional statements and the numeric target can be modelled as a fuzzy linguistic variable. In our study, we adapt and reinterpret several SD quality measures for this new framework and we test and analyse the adequacy of the different fuzzy operators involved. Moreover, we show the advantages of our perspective comparing our algorithm with others in the literature that are also based on fuzzy rules.

Acknowledgements

This work was partially supported by the R+D+i Project PID2020-113870GB-I00-"Desarrollo de herramientas de Soft Computing para la Ayuda al Diagnóstico Clínico y a la Gestión de Emergencias (HESOCODICE)", funded by MCIN/AEI/
10.13039/501100011033/. Raquel Fernandez-Peralta benefits from the fellowship FPU18/05664 granted by the Spanish Ministry of Science, Innovation and Universities within the Training University Lecturers (FPU) program. Balasubramaniam Jayaram would like to acknowledge the support obtained from SERB under the project MTR/2020/000506 for the work contained in this submission.

References

1. Martin Atzmueller. Subgroup discovery. *WIREs Data Mining and Knowledge Discovery*, 5(1):35–49, 2015.
2. Sumyea Helal. Subgroup discovery algorithms: A survey and empirical evaluation. *Journal of Computer Science and Technology*, 31:561–576, 2016.

On lattice structures on the set of Yager's implications

Isabel Aguiló[1,2], Vikash Kumar Gupta[3], Sebastia Massanet[1,2], Juan Vicente Riera[1,2], and Nageswara Rao Vemuri[3]

[1] Soft Computing, Image Processing and Aggregation (SCOPIA) Research Group, Dept. Mathematics and Computer Science, University of the Balearic Islands, Palma, Spain
{isabel.aguilo,s.massanet,jvicente.riera}@uib.es
[2] Health Research Institute of the Balearic Islands (IdISBa), Palma, Spain
[3] School of Mathematics and Statistics, University of Hyderabad 500 046, India
vikash52095@gmail.com, nrvemuriz@uohyd.ac.in

It is well known that the family of fuzzy implication functions is a complete distributive lattice with the point-wise operations maximum and minimum [1]. Furthermore, the family of continuous fuzzy implication functions is also a distributive lattice with the same operations, being a sublattice of the whole set of fuzzy implication functions. Recently, Vemuri et al. proposed in [2] the generation of continuous Archimedean t-norms through some operations that ensure that the set of continuous Archimedean t-norms is also a lattice.

Following some ideas presented in [2], in this contribution we prove that the set of Yager's f-generated implications, which was introduced in [3], is a distributive lattice with two binary operations that involve some continuous additive generators defined as the point-wise maximum and minimum of the additive generators of the input Yager's f-generated implications. Moreover, it is proved that the subfamilies of Yager's f-generated implications whose additive generators have finite or infinite value at 0, respectively, are sublattices. Finally, taking into account the equivalence of the family of Yager's f-generated implications whose generator has an infinite value at 0 with the family of Yager's g-generated implications whose generator has an infinite value at 1, some lattice operations are proposed within that family that involve the operations on the additive generators of continuous Archimedean t-conorms.

Acknowledgements

This work was partially supported by the R+D+i Project PID2020-113870GB-I00-"Desarrollo de herramientas de Soft Computing para la Ayuda al Diagnóstico Clínico y a la Gestión de Emergencias (HESOCODICE)", funded by MCIN/AEI/10.13039/501100011033/. All the authors thankfully acknowledge the financial support received by UoH-IoE Scheme under the grant UoH-IoE-RC2-21-015.

References

1. M. Baczyński and B. Jayaram. *Fuzzy Implications*, volume 231 of *Studies in Fuzziness and Soft Computing*. Springer, Berlin Heidelberg, 2008.
2. Nageswara Rao Vemuri, Balasubramaniam Jayaram, and Radko Mesiar. Generation of continuous t-norms through latticial operations. *Fuzzy Sets and Systems*, 2022. In Press. DOI=10.1016/j.fss.2022.09.005.
3. R.R. Yager. On some new classes of implication operators and their role in approximate reasoning. *Information Sciences*, 167:193–216, 2004.

Some generating methods of Interval-valued Fuzzy Implications

Vikash Kumar Gupta[1], Sebastia Massanet[2,3], and Nageswara Rao Vemuri[1]

[1] School of Mathematics and Statistics, University of Hyderabad 500 046, India
vikash52095@gmail.com & nrvemuriz@uohyd.ac.in

[2] Soft Computing, Image Processing and Aggregation (SCOPIA) research group,
Dept. of Mathematics and Computer Science, University of the Balearic Islands, 07122
Palma, Spain

[3] Health Research Institute of the Balearic Islands (IdISBa), 07010 Palma, Spain s.massanet@uib.es

Fuzzy implications are a generalization of classical implication to the multi- valued setting. Fuzzy implications are usually modelled on the unit interval $[0,1]$ as a mixed monotone functions whose restriction to $\{0,1\}$ coincide with the classical implication. For more details about fuzzy implications, we refer the readers to [1]. Fuzzy implications have been also generalized to the interval-valued setting and in this sense, interval-valued fuzzy implications (IVFIs) have been studied for quite a sometime. Though fuzzy implications on $[0,1]$ are well studied to some extent in the literature, so far, note that, IVFIs have not been explored much, except in some nascent works such as [2,3]. For instance, so far there do exist no generating methods of IVFIs that preserve various basic properties. Moreover, due to the applicational demand and theoretical significance, there is always a necessity for a comprehensive study of IVFIs.

Since the generating methods of fuzzy logic connectives lead to new fuzzy logic connectives that preserve some basic properties and also provide some insightful information of these connectives, in this work, we propose some generating methods of IVFIs and investigate the basic properties preserved by these methods. Especially, interesting is the generation method of an IVFI from two given fuzzy implications I and J on $[0,1]$ given by $K([a,b],[c,d]) = [I(a,c), I(a,J(b,d))]$ for all closed sub-intervals $[a,b],[c,d]$ of $[0,1]$. This method, which resembles the generation method $\circledast$ of fuzzy implications on $[0,1]$ proposed in [4] but differs on the second part of the interval, is deeply studied and showed that it preserves some basic desirable properties of IVFIs.

Acknowledgement

The authors acknowledge the financial support received by the Institute of Excellence (IoE) University of Hyderabad Scheme under the grant UoH-IoE-RC2-21-015. S. Massanet acknowledges also the partial support by the R+D+i Project PID2020-113870GB-I00-"Desarrollo de herramientas de Soft Computing para la Ayuda al Diagnóstico Clínico y a la Gestión de Emergencias (HESOCODICE)", funded by MCIN/AEI/10.13039/50110001 1033/.

References

1. Baczyński, M., Jayaram, B.: Fuzzy Implications. Studies in Fuzziness and Soft Computing, Vol. 231, Springer-Verlag, Heidelberg 2008.
2. Bedregal, B.C., Dimuro, G.P., Santiago, R.H.N., Reiser, R.H.S.: On interval fuzzy S-implications. Information Sciences 180 (2010), 1373–1389.
3. Cornelis, C., Deschrijver, G., Kerre, E.E.: Implication in intuitionistic fuzzy and interval-valued fuzzy set theory: construction, classification, application. International Journal of Approximate Reasoning (2004) 35 (1), 55-95.
4. Vemuri, N.R., Jayaram, B.: The $\circledast$ - composition of fuzzy implications: Closures with respect to properties, powers and families. Fuzzy Sets and Systems 275 (2015), 58-87.

On the monotonicity of Fuzzy Implications

Nageswara Rao Vemuri

School of Mathematics and Statistics, University of Hyderabad - 500 046, INDIA nrvemuriz@uohyd.ac.in

Fuzzy implications and triangular norms(t-norms) form an important pair that plays a significant role in fuzzy logic, approximate reasoning, decision making, control theory etc. They are defined as monotonous functions that satisfy some boundary conditions. For more details about these functions, please see [1,2]. Since, monotonicity is an important defining criterion for fuzzy logic connectives (FLCs), the researchers assumed a stronger version of monotonicity, which is known as strict monotonicity (SM) and explored it for FLCs like fuzzy negations and t-norms. In this regard, apart from the applicational role of (SM) of fuzzy negations and t-norms, we note the following theoretical importance of (SM):

- Fuzzy negations with (SM) and continuity play an important role in the characterization of (S, N)-implications, see Theorem 2.4.11 in [1].
- (SM) is an important property for the characterization of continuous Archimedean t-norms and consequently, continuous t-norms, see Theorem 3.43 in [2].
- (SM) of some unary functions on $[0, 1]$ is useful for defining some families of fuzzy implications which are called Yager's classes of fuzzy implications, see [1,3].

However, the property (SM) has not been proposed for fuzzy implications and a study similar to the above aspects is still pending. In this work, we propose (SM) for fuzzy implications and investigate the relationship between (SM) and other basic properties of fuzzy implications. Further, we show that fuzzy implications with the identity principle or the ordering property never satisfy (SM), which consequently show that no R-implication has the property (SM). Finally, we find the fuzzy implications with (SM) from different classes of fuzzy implications, mainly, (S, N)-implications, $QL-$implications and Yager's f and g-generated implications.

Acknowledgement

The authors acknowledge the financial support received by the UGC GoI under the grant No.F.30-513/2020(BSR).

References

1. Baczyński, M., Jayaram, B.: Fuzzy Implications. Springer-Verlag, Heidelberg 2008.
2. Klement, E.P., Mesiar, R., Pap, E.: Triangular Norms. volume 8 of Trends in Logic. Kluwer Academic Publishers, Dordrecht 2000.
3. Yager, R.R.: On some new classes of implication operators and their role in approximate reasoning. Information Sciences. 167, 193–216 (2004).

Part XII

SS10: New challenges and ideas in statistical inference and data analysis

Approximated Gibbs sampling for continuous fuzzy numbers

Antonio Calcagnì[1,2] and Przemyslaw Grzegorzewski[3,4]

[1] University of Padova, Italy
[2] GNCS Research Group, National Institute of Advanced Mathematics, Italy
[3] Warsaw University of Technology, Poland
[4] Systems Research Institute, Polish Academy of Sciences, Poland
antonio.calcagni@unipd.it
przemyslaw.grzegorzewski@pw.edu.pl

In many research contexts, statistical data analysis often requires dealing with several sources of uncertainty at the same time. This is the case, for instance, of analyzing data from social and economic surveys where both random (e.g., sample variation) and non-random (e.g., subjective responses) components are coupled together [1]. To disentangle different sources of uncertainty, epistemic fuzzy numbers can be used, and statistical methods can be generalized to deal with the fuzzy representation of the data. In this context, appropriate methods (e.g., fuzzy Expectation-Maximization) have been proposed to make estimation and other kinds of inference adequate. However, due to the way in which epistemic fuzzy estimators are built, they can suffer from excessive variance [4]. In this contribution, we propose to incorporate the general epistemic mechanism supposed to drive the generation of continuous fuzzy numbers in the definition of fuzzy estimators. The idea relies upon the use of a conditional probabilistic schema which links the parameters of fuzzy numbers (i.e., the non-random uncertainty) to the observed statistical model used for the data analysis. Thus, estimation and inference are performed using the Gibbs sampler-based approach, where the full conditional distribution is approximated by sampling from a quadratic approximation of the target posterior distribution [2,5]. A simulation study empirically demonstrates the speed and accuracy of the approximation across a wide range of conditions. Overall, the attempt is to provide a conditional sampling schema that aims at being general enough to cope with many empirical situations involving fuzzy data analysis.

References

1. Antonio Calcagnì, Niccolò Cao, Enrico Rubaltelli, and Luigi Lombardi. A psychometric modeling approach to fuzzy rating data. *Fuzzy Sets and Systems*, 447:76–99, 2022.
2. Daniele Durante, Francesco Pozza, and Botond Szabo. Skewed bernstein-von mises theorem and skew-modal approximations. *arXiv preprint arXiv:2301.03038*, 2023.
3. Paul E Green and Taesung Park. Bayesian methods for contingency tables using gibbs sampling. *Statistical Papers*, 45:33–50, 2004.
4. Przemyslaw Grzegorzewski and Joanna Goławska. In search of a precise estimator based on imprecise data. In *19th World Congress of the International Fuzzy Systems Association (IFSA), 12th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT), and 11th International Summer School on Aggregation Operators (AGOP)*, pages 530–537. Atlantis Press, 2021.
5. Jeffrey W Miller. Fast and accurate approximation of the full conditional for gamma shape parameters. *Journal of Computational and Graphical Statistics*, 28(2):476–480, 2019.

Part XIII

SS12: Representing and managing uncertainty: different scenarios, different tools

Multi-class classification based on interval modelling for datasets with large number of conditional attributes

Urszula Bentkowska, Wojciech Gałka, Marcin Mrukowicz, and Aleksander Wojtowicz

Institute of Computer Science, University of Rzeszów, Rzeszów, Poland {ubentkowska,wgalka,mmrukowicz}@ur.edu.pl aleksander.w220@gmail.com

A classification algorithm which uses interval modeling based on the notion of interval-valued fuzzy set [11] is proposed to cope with the problem of uncertainty [2,9]. The problem of k nearest neighborhood (k-NN) based multi-class classifier (cf. [5,8]) in the case of large number of conditional attributes in datasets is studied. This multi-class classifier is dedicated to datasets with large number of conditional attributes and it is tested on examples of high dimensional datasets which are microarrays. Such classification problems may be solved in diverse ways (cf. [1,4,10]). Here, an approach of creating the so called uncertainty intervals for the k-NN and then aggregating the obtained intervals with the use of interval-valued aggregation functions is used. Moreover, orders for intervals (cf. [3,7]) are applied to determine the decision class of an object. The applied interval-valued aggregation functions and interval orders are tested in terms of optimizing the performance of the considered classifier. Moreover, the quality of the proposed algorithm is compared with well-known decomposition methods for multi-class classification, such as OVO (one-versus-one) and OVR (one-versus-rest) (cf. [6]).

References

1. Abdullah, M.N., Yap, B.W., Sapri, N.N.F.F., Wan Yaacob, W.F.: Multi-class Classification for Breast Cancer with High Dimensional Microarray Data Using Machine Learning Classifier. In: Wah, Y.B., Berry, M.W., Mohamed, A., Al-Jumeily, D. (eds) Data Science and Emerging Technologies. DaSET 2022. Lecture Notes on Data Engineering and Communications Technologies, vol 165. Springer, Singapore (2023) https://doi.org/10.1007/978-981-99-0741-0-24
2. Bentkowska, U.: Interval-valued methods in Classifications and Decisions. Studies in Fuzziness and Soft Computing, vol. 378, Springer, Cham (2020)
3. Bustince, H., Fernandez, J., Kolesárová, A., Mesiar, R.: Generation of linear orders for intervals by means of aggregation functions. Fuzzy Sets Syst. **220** 69-77 (2013)
4. Do, T. -N., Poulet, F.: Classifying Very High-Dimensional and Large-Scale Multi-class Image Datasets with Latent-lSVM. In: 2016 Intl IEEE Conferences on Ubiquitous Intelligence and Computing, Advanced and Trusted Computing, Scalable Computing and Communications, Cloud and Big Data Computing, Internet of People, and Smart World Congress (UIC/ATC/ScalCom/CBDCom/IoP/SmartWorld), pp. 714-721. Toulouse, France (2016) doi: 10.1109/UIC-ATC-ScalCom-CBDCom-IoP-SmartWorld.2016.0116.
5. Elkano, M., Galar, M., Sanz, J., Lucca, G., Bustince, H.: IVOVO: A new interval-valued one-vs-one approach for multi-class classification problems, In: Joint 17th World Congress of International Fuzzy Systems Association and 9th International Conference on Soft Computing and Intelligent Systems (IFSA-SCIS), pp. 1-6. Otsu, Japan (2017) doi: 10.1109/IFSA-SCIS.2017.8023238
6. Galar, M., Fernández, A., Barrenechea, E., Bustince, H., Herrera, F.: An overview of ensemble methods for binary classifiers in multi-class problems: Experimental study on one-vs-one and one-vs-all schemes. Pattern Recognition **44** 1761-1776 (2011)
7. Karmakar, S., Bhunia, A.K.: A comparative study of different order relations of intervals. Reliab. Comput. **16** 38-72 (2012)
8. Ma, G., Lu, J., Liu, F. Fang, Z., Zhang, G.: Multiclass Classification With Fuzzy-Feature Observations: Theory and Algorithms. IEEE Transactions on Cybernetics (2022) doi: 10.1109/TCYB.2022.3181193.
9. Pękala, B.: Uncertainty Data in Interval-Valued Fuzzy Set Theory. Properties, algorithms and applications. Studies in Fuzziness and Soft Computing, vol. 367, Springer, Cham (2019)
10. Wei, T., Liu, W., Zhong, J., Gong, Y.: Multiclass Classification on High Dimension and Low Sample Size Data Using Genetic Programming. IEEE Transactions on Emerging Topics in Computing **10** (02), 704-718 (2022)
11. Zadeh, L.A.: The Concept of a Linguistic Variable and its Application to Approximate Reasoning-I. Information Sciences **8**, 199-249 (1975)

On the resolution of optimization problems subject to bipolar fuzzy relation equations

M. Eugenia Cornejo, David Lobo, and Jesús Medina

Department of Mathematics. University of Cádiz. Spain
mariaeugenia.cornejo, david.lobo, jesus.medina@uca.es

Optimization problems subject to bipolar fuzzy relation equations [1, 2] are currently being studied in some particular cases. To the best of our knowledge, in all published works, the outcome is an approximation of a solution of the optimization problem. In this paper, we present an alternative solving procedure for an optimization problem of the form

$$\begin{array}{ll} \text{Maximize} & f(x) \\ \text{s.t.} & (A^+ \circ x) \vee (A^- \circ \neg x) = b \end{array}$$

where f is order-preserving in the first k arguments and order-reversing in the remaining arguments, and $(A^+ \circ x) \vee (A^- \circ \neg x) = b$ is a bipolar fuzzy relation equation.

The developed procedure allows to compute exact solutions of the optimization problem instead of approximations. Furthermore, it is possible to obtain all the solutions of the problem instead of only one solution.

References

[1] M. E. Cornejo, D. Lobo, and J. Medina. On the solvability of bipolar max-product fuzzy relation equations with the standard negation. Fuzzy Sets and Systems, 410:1–18, 2021.

[2] M. E. Cornejo, D. Lobo, J. Medina, and B. De Baets. Bipolar equations on complete distributive symmetric residuated lattices: The case of a join-irreducible right-hand side. Fuzzy Sets and Systems, 442:92–108, 2022.

A comprehensive study of value reducts and bireducts *

M. Eugenia Cornejo, Fernando Chacón-Gómez, Jesús Medina, and Eloísa Ramírez-Poussa

Department of Mathematics. University of Cádiz. Spain
mariaeugenia.cornejo, fernando.chacon, jesus.medina, eloisa.ramirez@uca.es

In Rough Set Theory some mechanisms have been considered to reduce the size of datasets without loss of information. For instance, the notion of value reduct [2] arises to remove unnecessary information for a specific object, allowing a data analysis at an object level. On the other hand, bireducts [3] propose to reduce the sets of objects and attributes of a decision table simultaneously, obtaining different subtables without redundancies or contradictions. Hence, the use of both notions in data analysis provides a higher reduction of unnecessary data. This contribution is focused on the study of the relationship between bireducts and value reducts, facilitating the computation of ones from the others and viceversa [1]. For instance, in information tables, if a value reduct B for an object x is known, we can ensure the existence of a bireduct (X, B) with $x \in X$. On the other hand, given all the bireducts with a fixed subset of attributes B, for the objects belonging to all these bireducts it is possible to obtain value reducts B' with the property that $B' \subseteq B$.

References

1. F. Chacón-Gómez, M. E. Cornejo, J. Medina, and E. Ramírez-Poussa. Value reducts and bireducts: A comparative study. *Mathematical Methods in the Applied Sciences*, 46(2):1631–1650, 2023.
2. Z. Pawlak. *Rough Sets: Theoretical Aspects of Reasoning About Data*. Kluwer Academic Publishers, Norwell, MA, USA, 1992.
3. S. Stawicki, D. Ślęzak, A. Janusz, and S. Widz. Decision bireducts and decision reducts - a comparison. *International Journal of Approximate Reasoning*, 84:75–109, 2017.

* Partially supported by the 2014–2020 ERDF Operational Programme in collaboration with the State Research Agency (AEI) in project PID2019-108991GB-I00, with the Ecological and Digital Transition Projects 2021 of the Ministry of Science and Innovation in project TED2021-129748B-I00, and with the Department of Economy, Knowledge, Business and University of the Regional Government of Andalusia in project FEDER-UCA18-108612, and by the European Cooperation in Science & Technology (COST) Action CA17124.

On the Granular Representation of Fuzzy Quantifier-Based Fuzzy Rough Sets

Adnan Theerens and Chris Cornelis

Computational Web Intelligence, Department of Applied Mathematics, Computer Science and Statistics, Ghent University, Ghent, Belgium
{adnan.theerens, chris.cornelis}@ugent.be

Keywords: Fuzzy quantification · Fuzzy rough sets · Granular computing · Fuzzy integrals.

Fuzzy rough set theory can be used as a tool for dealing with inconsistent data when there is a gradual notion of indiscernibility between objects. It does this by providing lower and upper approximations of concepts. In this presentation, we will focus on some instances of the general class of fuzzy quantifier-based fuzzy rough sets [1,2] (FQFRS). In these models the lower and upper approximations are evaluated using binary and unary fuzzy quantifiers, respectively. One of the main targets of this study is to examine the granular representation of different models of FQFRS. We aim to determine if the lower and upper approximations of these models can be defined as unions of simple sets, i.e. fuzzy granules. These granules have a meaning in terms of "if ..., then ..." fuzzy rules, hence every set that can be represented granularly can be interpreted as a readable set of rules. These results have practical applications in the use of fuzzy rough approximations in rule induction algorithms: the granular representation of the lower approximation results in certain decision rules while the representation of the upper approximation results in possible rules. The main results of this presentation are the granular representation of Choquet-based fuzzy rough sets [3] and Sugeno-based fuzzy rough sets.

Acknowledgment

The research reported in this paper was conducted with the financial support of the Odysseus programme of the Research Foundation – Flanders (FWO). The grant number is G0H9118N.

References

1. A. Theerens, C. Cornelis, Fuzzy quantifier-based fuzzy rough sets, in: 2022 17th Conference on Computer Science and Intelligence Systems (FedCSIS), 2022, pp. 269–278. https://doi.org/10.15439/2022F231
2. A. Theerens, C. Cornelis, Fuzzy rough sets based on fuzzy quantification, arXiv preprint (2022) https://doi.org/10.48550/arXiv.2212.04327.
3. A. Theerens, O. U. Lenz, C. Cornelis, Choquet-based fuzzy rough sets, International Journal of Approximate Reasoning (2022). https://doi.org/10.1016/j.ijar.2022.04.006

A new algorithm for fuzzy rough rule induction with granular computing

Henri Bollaert[1], Chris Cornelis[1], Salvatore Greco[2], and Roman Słowiński[3]

[1] Department of Applied Mathematics, Computer Science and Statistics, Ghent University, Krijgslaan 281, S9, 9000 Ghent, Belgium {henri.bollaert, chirs.cornelis}@ugent.be
[2] Faculty of Economics, University of Catania emailsalgreco@unict.it
[3] Institute of Computing Science, Poznań University of Technology roman.slowinski@cs.put.poznan.pl

Keywords: fuzzy rough set theory · granular computing · rule induction · interpretable machine learning

Interpretability is the next frontier in machine learning research. In the search for white box models — as opposed to black box models, like random forests or neural networks — rule induction algorithms are a logical and promising option, since the rules can easily be understood by humans. Fuzzy and rough set theory have been successfully applied to this archetype, often separately. As both approaches to rule induction involve granular computing based on the concept of equivalence classes, it is natural to combine them. The QuickRules[2] algorithm was a first attempt at using fuzzy rough set theory for rule induction. It is based on QuickReduct, a greedy algorithm for building decision reducts. QuickRules already showed an improvement over other rule induction methods, but we can further improve it by using fuzzy rough sets based on ordered weighted average (OWA) operators[1], which reduces the impact of outliers, and by removing redundant rules in a pruning step, without decreasing the accuracy. To evaluate the full potential of a fuzzy rough rule induction algorithm, one needs, however, to start from the foundations. A fuzzy indiscernibility relation divides the data space into fuzzy granules that can be combined in a bottom up or a top-down way, each giving a successful rule set. One also needs to decide what kind of rule set is required, i.e., what is a minimal covering set of rules in this context, and is that desirable? Moreover, one should figure out how to decide which attributes each rule would preferably involve, and how to perform the inference process. Additionally, one should evaluate the effect of using OWA-based fuzzy rough sets on the performance of the algorithm and the number of rules. Finally, a study evaluating the effect of a particular definition of the indiscernibility relation should also be performed.

Acknowledgements

The research reported in this paper was conducted with the financial support of the Odysseus programme of the Research Foundation – Flanders (FWO). The grant number is G0H9118N. Henri Bollaert would like to thank the Special Research Fund of Ghent University (BOF-UGent) for funding his research.

References

1. Chris Cornelis, Nele Verbiest, and Richard Jensen. Ordered weighted average based fuzzy rough sets. In *Rough Set and Knowledge Technology*, volume 6401 of *Lecture Notes in Computer Science*, pages 78–85. Springer, 2010.
2. Richard Jensen, Chris Cornelis, and Qiang Shen. Hybrid fuzzy-rough rule induction and feature selection. pages 1151 – 1156, 09 2009.

Independent subcontexts in the multi-adjoint concept lattice framework

Roberto G. Aragón, Jesús Medina, and Eloísa Ramírez-Poussa

Department of Mathematics, University of Cádiz, Spain
{roberto.aragon,jesus.medina,eloisa.ramirez}@uca.es

In formal concept analysis, different methodologies have been developed to reduce the complexity of data processing [2,3]. One of these techniques is the factorization of formal contexts, which provides smaller contexts (subcontexts) easier to analyze. In [3], a method based on the operators of possibility theory was proposed for splitting a formal context with Boolean data into independent subcontexts. Recently, an initial study about the application of the (composition of) necessity operators, defined in possibility theory, in classical FCA and in the fuzzy multi-adjoint framework [3] was presented in [1]. In this work, we continue this previous study to extend the methodology given in [3] to the multi-adjoint framework. We address such a task by means of a Boolean relation defined from the fuzzy relation of a given context. Then, we detail under what conditions a (multi-adjoint) context can be factorized into independent subcontexts.

Acknowledgements Partially supported by the 2014–2020 ERDF Operational Programme in collaboration with the State Research Agency (AEI) in project PID2019-108991GB-I00, with the Ecological and Digital Transition Projects 2021 of the Ministry of Science and Innovation in project TED2021- 129748B-I00, and with the Department of Economy, Knowledge, Business and University of the Regional Government of Andalusia in project FEDER-UCA18-108612, and by the European Cooperation in Science & Technology (COST) Action CA17124.

References

1. R. G. Aragón, J. Medina, and E. Ramírez-Poussa. Study on the necessity operator to factorize formal contexts in a multi-adjoint framework. In D. Ciucci, I. Couso, J. Medina, D. Ślęzak, D. Petturiti, B. Bouchon-Meunier, and R. R. Yager, editors, *Information Processing and Management of Uncertainty in Knowledge-Based Systems*, Communications in Computer and Information Science, pages 107–117, Cham, 2022. Springer International Publishing.
2. R. Bělohlávek and V. Vychodil. Discovery of optimal factors in binary data via a novel method of matrix decomposition. *Journal of Computer and System Sciences*, 76(1):3–20, 2010.
3. D. Dubois and H. Prade. Possibility theory and formal concept analysis: Characterizing independent sub-contexts. *Fuzzy Sets and Systems*, 196:4–16, 2012.
4. J. Medina, M. Ojeda-Aciego, and J. Ruiz-Calviño. Formal concept analysis via multi-adjoint concept lattices. *Fuzzy Sets and Systems*, 160(2):130–144, 2009.

On the validity of attribute implications in concept lattices

M. Eugenia Cornejo, Jesús Medina, and Francisco José Ocaña

Department of Mathematics. University of Cádiz. Spain
mariaeugenia.cornejo, jesus.medina, franciscojose.ocana@uca.es

Many fuzzy generalizations of formal concept analysis have been proposed in the literature. Specifically, the fuzzy generalization of this theory within the multi-adjoint paradigm, based on adjoint triples, have been widely developed, giving rise to important advances in attribute and size reduction mechanisms [2,3,4,5]. A fundamental research line in formal concept analysis is the computation of relationships between attributes, called attribute implications. Attribute implications are presented as an alternative to obtain rules that model the given data set and contain the underlying information in it.

This paper will complement the work presented in [6], proposing a generalization of the usual definition of validity presented in the residuated concept lattice framework [1] to the multi-adjoint framework. Moreover, the monotonicity property of the operator associated with the validity will also be analyzed.

Acknowledgements

Partially supported by the 2014–2020 ERDF Operational Programme in collaboration with the State Research Agency (AEI) in project PID2019-108991GB-I00, with the Ecological and Digital Transition Projects 2021 of the Ministry of Science and Innovation in project TED2021-129748B-I00, and with the Department of Economy, Knowledge, Business and University of the Regional Government of Andalusia in project FEDER-UCA18-108612, and by the European Cooperation in Science & Technology (COST) Action CA17124.

References

1. R. Belohlavek and V. Vychodil. Attribute dependencies for data with grades il,. *International Journal of General Systems*, 45(7-8):864–888, 2016.
2. M. E. Cornejo, J. Medina, and E. Ramírez-Poussa. Attribute reduction in multi-adjoint concept lattices. *Information Sciences*, 294:41 – 56, 2015.
3. M. E. Cornejo, J. Medina, and E. Ramírez-Poussa. On the use of irreducible elements for reducing multi-adjoint concept lattices. *Knowledge-Based Systems*, 89:192–202, 2015.
4. M. E. Cornejo, J. Medina, and E. Ramírez-Poussa. Attribute and size reduction mechanisms in multi-adjoint concept lattices. *Journal of Computational and Applied Mathematics*, 318:388–402, 2017. Computational and Mathematical Methods in Science and Engineering CMMSE-2015.
5. M. E. Cornejo, J. Medina, and E. Ramírez-Poussa. Characterizing reducts in multi-adjoint concept lattices. *Information Sciences*, 422:364–376, 2018.
6. V. Liñeiro Barea, J. Medina, and I. Medina-Bulo. Towards generating fuzzy rules via fuzzy formal concept analysis. *7th European Symposium on Computational Intelligence and Mathematices*, pages 60–65, 2015.

Connecting Formal Concept Analysis Theories

M. José Benítez-Caballero and Jesús Medina

Universidad de Cádiz, Puerto Real (Cádiz), Spain mariajose.benitez,jesus.medina@uca.es

Formal Concept Analysis (FCA) and Rough Set Theory (RST) are two of the most important mathematical tools in order to manage and extract information stored in databases. FCA and RST can be connected in several ways, for example, the original representation of a given database is similar in both theories. They are also extended to the fuzzy setting, introducing different approaches.

One of the most general fuzzy frameworks of FCA is multi-adjoint concept lattices [3]. We also highlight an intermediate FCA framework between the fuzzy and crisp approaches, in this framework either the subsets of attributes or the subsets of objects remain crisp, meanwhile the other ones are treated as L-fuzzy sets, which is called generalized one-sided FCA [2]. Recently, it has been proven that this framework is a particular case of a multi-adjoint concept lattice in which a left-adjoint triple is considered [1].

On the other hand, multi-adjoint property-oriented and object-oriented concept lattices [4] was developed by blending the FCA and RST philosophy. Here, the conjunction and an implication of the adjoint triple assigned to each attribute are considered in order to define the forming-concept operators, which are called necessity and possibility operators.

The main goal of this paper is to develop a deep study of how these last theories are connected to FCA, taking special interest in the left-sided FCA framework, its translation into the property-oriented and object-oriented concept lattice frameworks, generalizing other existing works [5], and its application to the development of attribute reduction mechanisms.

Acknowledgements Partially supported by the 2014–2020 ERDF Operational Programme in collaboration with the State Research Agency (AEI) in project PID2019-108991GB-I00, with the Ecological and Digital Transition Projects 2021 of the Ministry of Science and Innovation in project TED2021-129748B-I00, and with the Department of Economy, Knowledge, Business and University of the Regional Government of Andalusia in project FEDER-UCA18-108612, and by the European Cooperation in Science & Technology (COST) Action CA17124.

References

1. Benítez-Caballero, M.J., Medina, J., Ramírez-Poussa, E.: Characterizing one-sided formal concept analysis by multi-adjoint concept lattices. Mathematics **10**(7) (2022). https://doi.org/10.3390/math10071020, https://www.mdpi.com/2227-7390/10/7/1020
2. Butka, P., Pócs, J.: Generalization of one-sided concept lattices. Computing and Informatics **32**(2), 355–370 (2013), http://www.cai.sk/ojs/index.php/cai/article/view/1625
3. Medina, J., Ojeda-Aciego, M., Ruiz-Calviño, J.: Formal concept analysis via multi-adjoint concept lattices. Fuzzy Sets and Systems **160**(2), 130–144 (2009)
4. Medina, J.: Multi-adjoint property-oriented and object-oriented concept lattices. Information Sciences **190**, 95–106 (2012). https://doi.org/10.1016/j.ins.2011.11.016
5. Shao, M.W., Li, K.W.: Attribute reduction in generalized one-sided formal contexts. Information Sciences **378**, 317 – 327 (2017). https://doi.org/http://dx.doi.org/10.1016/j.ins.2016.03.018, http://www.sciencedirect.com/science/article/pii/S0020025516301657

Index

ISBN: 978-84-09-52808-0